Hörmander, Lars

The null space of the $\overline{\partial}$-Neumann operator. (English) Zbl 1083.32033

Ann. Inst. Fourier 54, No. 5, 1305-1369 (2004).

In a first part, the author proves results concerning closed range of $\overline{\partial}$ and invertibility of $\Box$ (and concerning its null space on $(n-1)$-forms) on spherical shells, by systematic use of spherical harmonics. Some of these results are known, but the author's methods yield more precise constants.

In section 3, the author proves a theorem that precises the notion that when the Levi form has the critical signature at some point of the boundary ($(n-q-1,q)$), then the $\overline{\partial}$-Neumann problem is flawed on the domain at the level of $(0,q)$-forms: the dimension of the null space of $\Box_q$ is infinite, or $\overline{\partial}$ does not have closed range on $L^2_{(0,q)}$ (possibly both).

One of the results in section 2 is that the null space of $\Box_{n-1}$ has n independent multipliers. In section 4, it is shown that this property characterizes shells bounded by two confocal ellipsoids. A description of the null space of $\Box_{n-1}$ is obtained for these domains that allows to discuss the boundary behavior of the kernel of the orthogonal projection on this null space when the range of $\Box_{n-1}$ is closed, at a boundary point where the Levi form has signature $(0,n-1)$.

Reviewer: Emil J. Straube (College Station)

MSC:

32W05 $\overline{\partial}$ and $\overline{\partial}$-Neumann operators
32A25 Integral representations; canonical kernels (Szegő, Bergman, etc.)

Keywords:

$\overline{\partial}$-Neumann Laplacian; reproducing kernel

Full Text: DOI Numdam EuDML

References:

[1] H.P. Boas \& M.-C. Shaw, Sobolev estimates for the lewy operator on weakly pseudo-convex boundaries, Math. Ann274 (1986) p. 221-231 · Zbl 0588.32023

[2] L. Boutet de Monvel \& J. Sjöstrand, Sur la singularité des noyaux de Bergman et de szegö, Astérisque34-35 (1976) p. 123-164 · Zbl 0344.32010

[3] S.-S. Chern \& J.-K. Moser, Real hypersurfaces in complex manifolds, Acta Math133 (1974) p. 219-271 · Zbl 0302.32015

[4] S.-C. Chen \& M.-C. Shaw, Partial differential equations in several complex variables, AMS/IP Studies in advanced mathematics Vol. 19, Amer. Math. Soc, 2001 · Zbl 0963.32001

[5] L. Hörmander \(, L^2\) estimates and existence theorems for the \(\bar{⊠ }\) operator, Acta Math.113 (1965) p. 89-152 · Zbl 0158.11002

[6] L. Hörmander, The multinomial distribution and some Bergman kernels, Geometric analysis of PDE and several complex variables. Contemporary Mathematics Proceedings (to appear) · Zbl 1102.41028

[7] L. Hörmander, The analysis of linear partial differential operators I, Springer Verlag, 1983 · Zbl 0521.35001

[8] J.-J. Kohn, Harmonic integrals on strongly pseudo-convex manifolds. I., Ann. of Math.78 (1963) p. 112-148 · Zbl 0161.09302

[9] J.-J. Kohn \& L. Nirenberg, Non-coercive boundary problems, Comm. Pure Appl. Math18 (1965) p. 443-492 · Zbl 0125.33302

[10] J.-J. Kohn \& D.C. Spencer, Complex Neumann problems, Ann. of Math66 (1957) p. 89-140 · Zbl 0099.30605

[11] M.-C. Shaw, Global solvability and regularity for \(\bar{⊠ }\) on an annulus between two weakly pseudo-convex domains, Trans. Amer. Math. Soc291 (1985) p. 255-267 · Zbl 0594.35010

[12] M.-C. Shaw \(, L^2\) estimates and existence theorems for the tangential Cauchy-Riemann complex, Invent. Math82 (1985) p. 133-150 · Zbl 0581.35057

[13] L. Hörmander, The analysis of linear partial differential operators III, Springer-Verlag, 1985 · Zbl 0601.35001

[14] J.J. Kohn, Harmonic integrals on strongly pseudo-convex manifolds. II, Ann. Math. (2)79 (1964) p. 450-472 · Zbl 0178.11305