

**Lam, T. Y.**

**Serre's problem on projective modules.** (English) [Zbl 1101.13001](#)  
[Springer Monographs in Mathematics](#). Berlin: Springer (ISBN 3-540-23317-2/hbk). xxi, 401 p. (2006).

In his famous 1955 FAC paper, J.-P. Serre asked if all finitely generated projective  $k[X_1, \dots, X_n]$ -modules are free when  $k$  is a field. This question became known as "Serre's conjecture" and was finally answered affirmatively and independently by Quillen and Suslin in 1976. This book is a greatly expanded and updated version of the author's (now out of print) "Serre's Conjecture" [Lect. Notes Math. 635 (1978; [Zbl 0373.13004](#))]. Chapters 1–5 and 7 of the present volume correspond to the six chapters of the Lecture Notes volume. Besides the many updates and revisions here and there, the totally new additions are chapter 6 on  $K_1$ -analogs of Serre's conjecture and chapter 8 on new developments since 1977.

The book starts with the basics of projective modules and the  $K_0$  and  $K_1$  groups, and then gives the classical, partial results about Serre's conjecture. It then gives careful treatments of the solution by unimodular rows and the solution using Horrock's theorem and Quillen's patching theorem. Several analogs of Serre's conjecture are also discussed. The first seven chapters are self-contained with all proofs and definitions, and assume only a beginning graduate course in algebra. The final chapter concerning the new developments (since 1977) is more expository in nature and does not include proofs. However, it has over 600 references. This well-written book is the definitive treatment of "Serre's conjecture" – its history, solution, and generalizations – and will be of interest to both beginning graduate students and advanced researchers in this field.

Reviewer: [David F. Anderson \(Knoxville\)](#)

**MSC:**

- [13-02](#) Research exposition (monographs, survey articles) pertaining to commutative algebra
- [13C10](#) Projective and free modules and ideals in commutative rings
- [13F20](#) Polynomial rings and ideals; rings of integer-valued polynomials
- [13A99](#) General commutative ring theory

Cited in **3** Reviews  
Cited in **82** Documents

**Keywords:**

[Serre's conjecture](#); [unimodular rows](#); [Quillen's patching theorem](#); [Horrock's theorem](#)