Dodds, P.; **de Pagter, B.**; **Sedaev, A.**; **Semenov, E.**; **Sukochev, F.**
Singular symmetric functionals. (English. Russian original) Zbl 1090.46020
J. Math. Sci., New York 124, No. 2, 4867-4885 (2004); translation from Zap. Nauchn. Semin. POMI 290, 42-71 (2002).

The inspiration for the results of this paper is the theorm of *J. Dixmier* [C. R. Acad. Sci., Paris, Sér. A 262, 1107–1108 (1966; Zbl 0141.12902)] that there exist nonnormal traces on the von Neumann factor $B(H)$ which are singular in the sense that they vanish on all finite rank operators. In a previous paper [Positivity 2, No. 1, 47–75 (1998; Zbl 0915.46021)] by the authors (except A. Sedaev), it was shown that Dixmier's methods are related to ones used in the theory of (commutative) symmetric Banach function spaces by observing that Dixmier's nonnormal traces resemble continuous linear functionals on certain Marcinkiewicz operator ideals that are called symmetric. The present paper is a continuation of this line of investigation in presenting a new construction of singular symmetric linear functions. It is based on the important notion of "almost convergence" introduced by *G. G. Lorentz* [Acta Math., Uppsala 80, 167–190 (1948; Zbl 0031.29501)].

Because of the highly technical nature of the material, we merely list the titles of its sections with comments. Following the Introduction, Section 2, "Definitions and Preliminaries", contains the definition of what is meant by a symmetric Banach function space and, in particular, a Marcinkiewicz space. Section 3, "Singular Symmetric Functionals defined by Banach Limits", deals with the construction of symmetric linear functionals on Marcinkiewicz spaces, using the results of the theory of translation invariant linear functionals on $\ell^\infty(\mathbb{N})$. In the final Section 4, "Seminorms defined by Singular Symmetric Functionals", an interesting result involving Orlicz spaces is presented.

Reviewer: W. A. J. Luxemburg (Pasadena)

MSC:

46E30 Spaces of measurable functions (L^p-spaces, Orlicz spaces, Köthe function spaces, Lorentz spaces, rearrangement invariant spaces, ideal spaces, etc.)
46L52 Noncommutative function spaces

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symmetric Banach function space; Marcinkiewicz space; singular symmetric functionals

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