

Leistner, Thomas

Holonomy and parallel spinors in Lorentzian geometry. (English) Zbl 1088.53032

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The author studies Lorentzian manifolds and their holonomy groups under the condition that they have parallel spinors. Firstly, he shows that a semi-Riemannian product manifold admits a parallel spinor if and only if every factor admits a parallel spinor. Using the Wu-decomposition theorem and the vector field associated to the parallel spinor, he solves the existence problem of parallel spinors for Lorentzian manifolds which decompose into irreducible factors. For indecomposable, non-irreducible Lorentzian manifolds, their holonomy group is contained in the parabolic group $(\mathbb{R}^* \times SO(n)) \times \mathbb{R}^n$; and it is proved that the $SO(n)$ -component of the holonomy group is the holonomy group of a certain metrical vector bundle which is called the screen bundle. The existence of parallel spinors implies that the \mathbb{R}^* -component of the holonomy has to be trivial and the $SO(n)$ -component cannot be Abelian. If the latter contains $SU(n/2)$, then the existence of a parallel spinor implies that it is equal to $SU(n/2)$.

The author studies the following question: Which Lie groups can occur as $SO(n)$ -component of the holonomy group of a simply connected indecomposable, non-irreducible Lorentzian manifold? He obtains an algebraic criterion, based on the first Bianchi identity, which is analogous to the Berger criterion, but restricted to the $so(n)$ -component; and he proves that if G is the $SO(n)$ -component of an indecomposable, non-irreducible Lorentzian manifold, then it is a Riemannian holonomy group, provided that it acts irreducibly and is simple, or provided that $G \subset U(n/2)$. Therefore, an indecomposable Lorentzian manifold with parallel spinor vector field does not have a holonomy group of coupled type.

Reviewer: [Marisa Fernandez \(Bilbao\)](#)

MSC:

- [53C29](#) Issues of holonomy in differential geometry
- [53C50](#) Global differential geometry of Lorentz manifolds, manifolds with indefinite metrics
- [53C27](#) Spin and Spin^c geometry

Cited in 1 Review Cited in 11 Documents
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Keywords:

[Lorentzian manifolds](#); [holonomy groups](#); [parallel spinors](#); [Walker coordinates](#); [Berger criterion](#); [screen bundle](#)