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Perspectives on A -homotopy theory and its applications. (English) Zbl 1082.37050
Discrete Math. 298, No. 1-3, 39-61 (2005).

This article is a survey of a combinatorial homotopy theory, the A -theory, which concerns simplicial complexes and graphs. In the first section, the authors recall the definition of this homotopy theory in the two frameworks [see *H. Barcelo, X. Kramer, R. Laubenbacher and C. Weaver*, Adv. Appl. Math. 26, 97–128 (2001; [Zbl 0984.57014](#))]. The theory has some similarities with the classical homotopy theory of a pointed topological space. For instance, the Seifert-van Kampen theorem for the fundamental group is valid and the higher-dimensional groups are abelian too. It is however different, as a contractible complex may have nontrivial A -groups, and there is no invariance under triangulation. The authors give also an algorithm for computing the abelianization of the A_1 -groups. Finally, the last sections contain some links and applications of the A -theory, like, e.g., homotopy theory of matroid complexes (S. B. Maurer), graph theory (L. Lov asz) or subspace arrangement (E. Babson, H. Barcelo, R. Laubenbacher).

Reviewer: [Christophe Dupont \(Orsay\)](#)

MSC:

[37F20](#) Combinatorics and topology in relation with holomorphic dynamical systems

Cited in **2** Reviews
Cited in **13** Documents

Keywords:

A -theory; combinatorial homotopy; simplicial complexes; graphs

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