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Hyperbolic volume of representations of fundamental groups of cusped 3-manifolds. (English)

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A complete hyperbolic metric on a compact or cusped 3-manifold M induces a discrete faithful representation $\rho : \pi_1(M) \to \mathrm{PSL}(2, \mathbb{C}) = \mathrm{Isom}(\mathbb{H}^3)$: namely, one identifies the universal cover $\widetilde{M}$ of M with $\mathbb{H}^3$ and $\pi_1(M)$ with the covering translations of the cover $\mathbb{H}^3 = \widetilde{M} \to M$. Mostow rigidity then implies that ρ is well-defined up to conjugation within $\mathrm{PSL}(2, \mathbb{C})$, hence that the (finite) volume $\mathrm{vol}(\rho) = \mathrm{vol}(M)$ of M is a topological invariant.

The author notes that Dunfield has explained how to calculate a volume $\mathrm{vol}(\rho)$ for all representations $\rho : \pi_1(M) \to \mathrm{PSL}(2, \mathbb{C})$, discrete and faithful or not, provided that the cusp structure is respected.

This paper is devoted to showing that the Dunfield volume $\mathrm{vol}(\rho)$ is well-defined – that is, $\mathrm{vol}(\rho)$ is independent of the choices made in Dunfield's construction. The volume can be computed by "straightening" any ideal triangulation of M.

The author also proves that, if M is cusped-hyperbolic and $\mathrm{vol}(\rho) \geq \mathrm{vol}(M)$, then, in fact, $\mathrm{vol}(\rho) = \mathrm{vol}(M)$ and ρ is discrete and faithful.

Reviewer: James W. Cannon (Provo)

MSC:

57M50	General geometric structures on low-dimensional manifolds
53C24	Rigidity results
53A35	Non-Euclidean differential geometry
57N10	Topology of general 3-manifolds (MSC2010)

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hyperbolic volume; cusped 3-manifold; $\mathrm{PSL}(2, \mathbb{C})$

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