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Benford's law, recurrence relations and equidistributed series. (Loi de Benford, relations de récurrence et suites équidistribuées.) (French) Zbl 1084.11005

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The first digit law or Benford's law states that if we randomly select numbers from listings, tables of statistics, etc., the proportion of numbers with leading digit equals to d is close to $\log_{10}(1+1/d)$ against the naïvely expected value 1/9. More generally, given a positive integer $b \geq 2$, we say that a sequence $(a_n)_{n\geq 0}$ of real numbers larger than 1 satisfies Benford's law with respect to the base b if for every integer d, $1 \leq d \leq b-1$, the leading digit in the b-adic expansion of a_n is equal to d with frequency $\log_b(1+1/d)$. It is an obvious fact that every sequence of real numbers larger than 1 satisfies Benford's law with respect to the base 2.

Some classical sequences of positive integers satify this law. For instance, this is the case of the Fibonacci sequence. The present paper is motivated by this latter result and its main contribution reads as follows. Let $p(x) = x^q - c_1x^{q-1} - \ldots - c_q$ be a polynomial with real coefficients having a real simple root $\xi > 1$ such that ξ is larger than the modulus of every other root of p. Let $(a_n)_{n\geq 0}$ be a sequence of real numbers larger than 1 and satisfying the following recurrence:

$$a_{n+q} = c_1a_{n+q-1} + \ldots + c_qa_n.$$

Assume moreover that

$$\inf\left\{\frac{a_n}{\xi^n},\ \ n \geq 0\right\} > 0.$$

Then, for every $b \geq 3$ such that $\log_b(\xi)$ is irrational, the sequence $(a_n)_{n\geq 0}$ satisfies Benford's law with respect to the base b.

The proof relies on Weyl's criterion for a sequence of real numbers to be uniformly distributed modulo one. This paper provides a nice introduction to Benford's law and Weyl's criterion.

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MSC:

11B37 Recurrences
11K31 Special sequences
11J71 Distribution modulo one

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Benford's law; Weyl's criterion; linear recurrences

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