Qin, Huizeng; **Lu, Youmin**

Application of uniform asymptotics method to analyzing the asymptotic behaviour of the general fourth Painlevé transcendent. (English) Zbl 1090.34072

Int. J. Math. Math. Sci. 2005, No. 9, 1421-1434 (2005).

The authors use the uniform asymptotic method proposed by *A. P. Bassom*, *P. A. Clarkson*, *C. K. Law* and *J. B. McLeod* [Arch. Rat. Mech. Anal., 143, 241–271 (1998; Zbl 0912.34007)] to study the general solution of the fourth Painlevé equation

$$y'' = \frac{y'^2}{2y} + \frac{2}{3}y^3 + 4xy^2 + 2(x^2 - \alpha)y + \frac{\beta}{y}. \tag{P_{IV}}$$

At the present time, there are not many results about the asymptotics of fourth Painlevê equation (see the second author [Int. J. Math. Math. Sci. 2003, No. 13, 845–851 (2003; Zbl 1025.34091)]). The authors study the behaviour of the real solutions of (P_{IV}) when $\beta > 0$ and $\alpha > 0$, and obtain the following result on the asymptotics of its real solutions.

Theorem: If $\beta > 0$, then the solutions of (P_{IV}) cannot cross the x-axis. Furthermore, if $\alpha > 0$, then the only negative solution of Painlevé equation (P_{IV}) that does not blow up at any finite point when x goes to positive infinity is oscillating as $x \to +\infty$ and it satisfies the following relations:

As $x \to +\infty$,

$$\begin{aligned} y &= -\frac{2}{3}x \pm d\cos\phi + O(x^{-1}), \quad x \to +\infty \\ y' &= \frac{2\sqrt{3}x}{3} d\sin\phi + O(x^{-1}), \quad x \to +\infty, \end{aligned} \tag{1}$$

where $\phi = (\sqrt{3}/3)x^2 - (\sqrt{3}/4)d^2 \log x + \phi_0 + O(x^{-1}), d$ and ϕ_0 are real parameters.

(2)As $x \to -\infty$, y blows up at a finite point of x.

Reviewer: Nikolay Vasilye Grigorenko (Kyïv)

MSC:

34M55 Painlevé and other special ordinary differential equations in the complex domain; classification, hierarchies

Cited in **1** Document

Keywords:

Painlevê equations; asymptotics

Full Text: DOI EuDML