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Representation of manifolds. (English) JFM 54.0611.01

Math. Ann. 100, 552-578 (1928).

Der *Brouwersche Abbildungsgrad*, der ursprünglich für stetige Abbildungen von *simplizialen Mannigfaltigkeiten* definiert wurde, wird hier allgemeiner für abgeschlossene "topologische" Mannigfaltigkeiten (Dieser Ausdruck rührt von *H. Hopf* [vgl. das folgende Referat] her. Verf. verwendet hierfür die Bezeichnung: "lokal simpliziale Mannigfaltigkeit") erklärt, d. h. für kompakte zusammenhängende topologische Räume, die ein System von Umgebungen besitzen, deren jede mit dem Innern einer Euklidischen n-dimensionalen Kugel homöomorph ist. Die Frage, ob sich derartige Räume "triangulieren lassen", also selbst simpliziale Mannigfaltigkeiten sind, ist bekanntlich offen. Die wesentlichen Eigenschaften des Abbildungsgrades – in erster Linie die Multiplikation der Abbildungsgrade bei Zusammensetzung von mehreren Abbildungen – bleiben bei dieser Verallgemeinerung erhalten.

Reviewer: Hurewicz, W., Dr. (Amsterdam)

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References:

[1] Brouwer, ??ber Abbildung von Mannigfaltigkeiten?, Math. Annalen 71, p. 97; this work is referred to in the sequel as ?A. v. M.?.

[2] ?A. v. M.?, p. 97. By ann-dimensional element is understood the topological image of ann-dimensional simplex and by a simplex star ofR n a finite number of simplexes dense in a neighbourhood of a common vertexO, no two of which have inner points in common and any two of which have a commonp-dimensional face (0?p?n?1) but no further common point.

[3] ?Beweis der Invarianz desn-dimensionalen Gebiets?, Math. Annalen71, p. 305 and 306; the definition there given is here completed in accordance with a verbal communication from Prof. Brouwer.

[4] ?A. v. M.?, p. 97-98.

[5] ?A. v. M.?, p. 100.

[6] ?A. v. M.?, p. 101.

[7] Hausdorff, ?Grundz?ge der Mengenlehre? (1914), p. 213; this work is referred to in the sequel by ?Hausdorff?.

[8] Hausdorff, ?Grundz?ge der Mengenlehre? (1914), p. 260.

[9] Such manifolds have been defined by Weyl, ?Die Idee der Riemannschen Fl?che? (1913), p. 17-18; Tibor Rad?, ??ber den Begriff der Riemannschen Fl?che?, Acta litt. scient. Reg. Univ. Franc. Jos., 2, Fasc. II (1925), who proves that a closed, locally simplicial, 2-dimensional manifold is simplicial; and v. Ker?kj?rt?, ?Vorlesungen ?ber Topologie I? (1923), Einleitung, p. 5-6.

[10] These remarks have been suggested to me by Prof. Brouwer.

[11] ?Zum Metrisationsproblem?, Math. Annalen94, p. 310, Hauptsatz; extended by Tychonoff to regular spaces, Math. Annalen 95.

[12] ?A. v. M.?, p. 106, Satz 1; in the above terminology this theorem may be written: If a closed, two-sided,n-dimensional, simplicial manifold ? be uniquely and continuously represented on a simplicialn-dimensional manifold ??, there exists a finite whole numberc, invariant under continuous modification of the representation, with the property that the image of ? covers every region of ?? altogetherc times positively; if ?? be one-sided or openc is always zero. This numberc is called the degree of the representation.

[13] As with simplicial manifolds, Brouwer, ?A. v. M.?, p. 100, assign to each element of a locally simplicial manifold, a regular Euclidean simplex of fixed length of edge as its ?representative simplex?, and let there be a topological correspondence between the element and its representative simplex; then by a segment, segment path, component simplex, (n-1)-dimensional simplex inE?, is understood the image of a segment, segment path, component simplex, (n-1)-dimensional simplex respectively, in the representative simplex ofE?.

[14] Brouwer, ?Beweis desn-dimensionalen Jordanschen Satzes?, Math. Annalen71, p. 317, footnote.

[15] Brouwer ?Beweis desn-dimensionalen Jordanschen Satzes?, Math. Annalen71, p. 314.

[16] By an (n-1)-dimensional sphere ofE? is understood the image inE? of an (n-1)-dimensional sphere in the representative simplex ofE?.

[17] ?A. v. M.?, p. 101.

[18] ?A. v. M.?, p. 100.

[19] ?A. v. M.?, p. 108. The indicatrix of the (n-1)-dimensional simplexA 1 A 2 ...A n considered as a face of the simplexA 1 A 2...A n+1 is defined to beA 1,A 2, ...,A n whereA 1,A 2, ...,A n ,A n+1 is the indicatrix ofA 1 A 2...A n+1

[20] Brouwer, ??ber Jordansche Mannigfaltigkeiten?, Math. Annalen71, p. 323, ? 4.

[21] Brouwer, ??ber Jordansche Mannigfaltigkeiten?, Math. Annalen71, p. 323, ? 4, Satz 4; we have above used a particular case of this theorem.

[22] Such an element exists whenE? andE? have inner points in common Hausdorff, Axiom (B).

[23] We are here using Brouwer's generalized indicatrix, ??ber Jordansche Mannigfaltigkeiten?, p. 324, ? 5; for the extension of the indicatrix conception to locally simplicial manifolds 1 am indebted to Prof. Brouwer personally.

[24] See remark 2 above.

[25] ?A. v. M?, p. 101-105.

[26] W. Wilson, ?Representation of a simplicial manifold on a locally simplicial manifold?, Amsterdam Proceedings29 (1926), p. 1129 sqq.; for the leading idea of the proof there given the writer was indebted to a remark on Prof. Brouwer.

[27] ?A. v. M.?, p. 106.

[28] ?A. v. M.?, p. 106, Satz 1; in the above terminology this theorem may be written: If a closed, two-sided,n-dimensional, simplicial manifold ? be uniquely and continuously represented on a simplicialn-dimensional manifold ??, there exists a finite whole numberc, invariant under continuous modification of the representation, with the property that the image of ? covers every regioni of ?? altogetherc times positively; if ?? be one-sided or openc is always zero see footnote13) above.

[29] Tibor Rad?, loc. cit. ??ber den Begriff der Riemannschen Fl?che?, Acta litt. scient. Reg. Univ. Franc. Jos., 2, Fasc. II (1925), Hilfsatz 1; see also remark 2 in the introduction.

[30] That is,V i (2) is a simplex ofE 2 with the same vertices asV i ; we recall (footnote14) as with simplicial manifolds, Brouwer, ?A. v. M.?, p. 100, assign to each element of a locally simplicial manifolds, a regular Euclidean simplex of fixed length of edge as its ?representative simplex?, and let there be a topological correspondence between the element and its representative simplex; then by a segment, segment path, component simplex, (n-1)-dimensional simplex inE?, is understood the image of a segments, segment path, component simplex, (n-1)-dimensional simplex respectively, in the representative simplex ofE?. above) that a simplex ofE 2 is the image inE 2 of a simplex in the representative simplex ofE 2.

[31] By the boundary of a set of simplexes among which the incidence relations are assigned is understood the set of those (n-1)-dimensional faces which are incident with only one simplex.

[32] Use is being here made of the ?gemischte Zerlegung? of Brouwer, ?Erweiterung des Definitionsbereichs einer stetigen Funktion?, Math. Annalen79, p. 210.

[33] This multiplication of elements was suggested by the duplication of elements used by Brouwer. ?Transformations of Projective Spaces?, Amsterdam Proceedings29 (1926), No. 6.

[34] Since the values ofp j ?q j on different sides ofF differ by unity, and on that side ofF not inX j both numbersp andq are zero.

[35] Such a chain of simplexes shall be referred to briefly as a chain of simplexes or merely as a chain.

[36] By \{U i (k) \} is understood the set of all simplexesU i (k) ,i. e., the suffixi takes all values for which the vertices ofU i are all inE k ;e. g. in \{U i (1) \} the summation is extended over a different set of the suffixesi from that in \{U i (2) \}; similarly with \{V i (k) \}.

[37] This is the ?regular subdivision? of Veblen, Analysis Situs, p. 85-86.

[38] In the sense of Hausdorff, ?Grundz?ge der Mengenlehre?, p. 260-261

[39] By \{U ij (1) \} is understood the set of all component simplexes of \{U i (1)\}, and, as in previous paragraphs, by \{U i (1) \} the set of all existing simplexesU i (1) ; similarly for \{V ij (1) \}.

[40] As in ? 9, by \{1 U ij (2) \} is understood the set of all component simplexes of \{1 U i (2) \}; similarly for \{1 V ij (2) \}.

[41] ??ber Jordansche Mannigfaltigkeiten?, Math. Annalen71, p. 320, ?? 5 and 6.

[42] Brouwer, ??ber Jordansche Mannigfaltigkeiten?, p. 324 and the remarks on p. 598.

[43] Brouwer, op. cit. ??ber Jordansche Mannigfaltigkeiten?, p. 324, footnote.

[44] Brouwer, op. cit. ??ber Jordansche Mannigfaltigkeiten?, Satz 6.