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L^p–L^q-estimates for singular integral operators arising from hyperbolic equations. (English)
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Partial diff. Equ., Berkeley 1971, Proc. Sympos. Pure Math. 23, 479-481 (1973).

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Recently, *R. S. Strichartz* [Trans. Am. Math. Soc. 148, 461–471 (1970; Zbl 0199.17502)] has obtained the estimate

$$\|u(\cdot t)\|_q < M(t)\|u_t(\cdot 0)\|_p \tag{*}$$

for solutions to $\Box u = 0$ vanishing at $t = 0$, for certain $p \leq 2 \leq q$. This estimate is used [*R. S. Strichartz*, J. Funct. Anal. 5, 218–235 (1970; Zbl 0189.40701)] to derive estimates for the corresponding non-homogeneous equation, yielding existence and uniqueness theorems for the Cauchy problem for linear and nonlinear variations of the wave equation, and then to obtain some results on the wave operator for $\Box u = H(u)$. The proof of $(*)$ depended on the spherical symmetry of $\Box$, hence does not carry over easily to more general operators.

In the present paper L^p–L^q-estimates are derived for singular integral operators whose kernels have singularities on smooth $n-1$ dimensional surfaces. Results analogous to $(*)$ are then obtained for more general hyperbolic operators, yielding corresponding applications. Sufficient conditions are given for $|\xi|^{-\alpha}e^{i\,|\xi|\Psi(\xi)}\varphi(\xi)$ to be a multiplier from L^p to L^q_{loc}, where φ and Ψ are homogeneous of degree zero. Proofs are sketched.

Reviewer: Walter Littmann

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MSC:

42B25 Maximal functions, Littlewood-Paley theory
35L30 Initial value problems for higher-order hyperbolic equations
42A45 Multipliers in one variable harmonic analysis