Ambrosetti, Antonio; Rabinowitz, Paul H.
Dual variational methods in critical point theory and applications. (English) Zbl 0273.49063
J. Funct. Anal. 14, 349-381 (1973).

Consider the nonlinear elliptic partial differential equation

$$L(u) \equiv -\sum_{i,j=1}^{n} (a_{ij}(x)u_{x_i})_{x_j} + c(x)u = p(x,u), \quad x \in \Omega,\ u = 0,\ x \in \partial\Omega, \tag{*}$$

where $\Omega \subset \mathbb{R}^n$ is a smooth bounded domain. Formally, the critical points of the functional

$$I(u) = \int_\Omega \left[\frac{1}{2} \sum_{i,j=1}^{n} (a_{ij}(x)u_{x_i})_{x_j} + c(x)u^2 - P(x,u(x)) \right] dx,$$

where $P(x,u)$ is a primitive of $p(x,u)$, are solutions of (*). The authors construct dual variational methods to enable them to prove the existence and estimate the number of critical points possessed by a real continuously differentiable functional on a real Banach space, and then apply their results to various existence problems for equations of type (*). They also apply them to problems with linear term added, i.e.

$$L(u) = a(x)u + p(x,u), \quad x \in \Omega;\ u = 0,\ x \in \partial\Omega,$$

as well as to nonlinear integral equations of the form

$$v(x) = \int_\Omega g(x,y)q(y,v(y))\,dy.$$

Reviewer: H. S. P. Grässer

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MSC:

58E05 Abstract critical point theory (Morse theory, Lyusternik-Shnirel'man theory, etc.) in infinite-dimensional spaces
35J20 Variational methods for second-order elliptic equations

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