Jacobi, C. G. J.

A few remarks on cubic residues. (De residuis cubicis commentatio numerosa.) (Latin)

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J. Reine Angew. Math. 2, 66-69 (1827).

Jacobi remarks that Gauß has announced a memoir on biquadratic residues, in which he will prove a criterion for 2 to be a biquadratic residue of primes p. He observes that for cubic residuacity of primes $p = 3n+1$ one has to consider the representations $4p = L^2 + 27M^2$. He announces the following theorem: If p and q are prime numbers of the form $3n+1$, with $4p = L^2 + 27M^2$, and if x is an integer with $x^2 + 3 \equiv 0 \pmod{q}$, then q will be a cubic residue with respect to p if and only if $\frac{L+3mx}{L-3Mx}$ is a cubic residue with respect to p.

His second theorem deals with primes $q = 6n-1$; he calls integers x with $x^{\frac{q+1}{3}} \equiv 1 \pmod{p}$ cubic residues of q, and announces the following result: If p is a prime of the form $6n+1$, if $4p = L^2 + 27M^2$, and if q is a prime of the form $6n-1$, then q will be a cubic residue with respect to p if and only if $\frac{L+3m\sqrt{-3}}{L-3M\sqrt{-3}}$ is a cubic residue with respect to p.

In addition he remarks that if $p = 3n+1$ satisfies $4p = L^2 + 27M^2$, then L is the minimal remainder modulo p of $-\frac{(n+1)(n+2)\cdots 2n}{1\cdot 2\cdots n} = -\binom{2n}{n}$ that has the form $3k+1$, and gives a similar result for primes $p = 7n+1$.

Jacobi presented the proofs of these results in his lectures; see [*F. Lemmermeyer* (ed.) and *H. Pieper* (ed.), Vorlesungen über Zahlentheorie. Carl Gustav Jacob Jacobi, Wintersemester 1836/37, Königsberg. Augsburg: ERV Dr. Erwin Rauner Verlag (2007; Zbl 1148.11003)].

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