Kadison, Richard V.

Isometries of operator algebras. (English) Zbl 0045.06201

Ann. Math. (2) 54, 325-338 (1951).

Let X and Y be compact Hausdorff spaces, and let $C(X, K)$ resp. $C(Y, K)$ denote the sets of continuous complex-valued functions on X resp. Y. It is known that a linear isometry of $C(X, K)$ onto $C(Y, K)$ (under the uniform metric for both spaces) is actually an algebraic isomorphism followed by multiplication by a function in $C(Y, K)$ which has absolute value 1 [*M. H. Stone*, Trans. Am. Math. Soc. 41, 375–481 (1937; Zbl 0017.13502)]. A non-commutative analogue of this theorem is given here, dealing with a certain class of not necessarily commutative Banach algebras (= normed rings). A C^*-algebra is a Banach algebra admitting an adjoint operation $x \to x^*$ satisfying axioms 1')–6') of *I. Gel'fand* and *M. Neĭmark* [Mat. Sb., N. Ser. 12(54), 197–213 (1943; Zbl 0060.27006)]. Gel'fand and Neĭmark (loc. cit., Theorem 1) have shown that every C^*-algebra is algebraically, normwise, and adjoint-preserving isomorphic to a uniformly closed algebra of bounded operators on some Hilbert space. Thus the author can, in discussing C^*-algebras, consider only algebras of operators.

Two preliminary results are obtained first: the extreme points of the unit sphere in a C^*-algebra $\mathfrak{A}$ are the set of partially isometric operators $U \in \mathfrak{A}$ where $U^*U = E$, $UU^* = F$, and $(I - F)\mathfrak{A}(1 - E) = 0$; the positive part of the unit sphere of $\mathfrak{A}$ has as extreme points the projections in $\mathfrak{A}$.

Turning next to isometries of C^*-algebras, the author proves: an isomorphism of a C^*-algebra $\mathfrak{A}$ which preserves the *-operation is isometric and preserves commutativity; an isometric linear mapping ρ of a C^*-algebra $\mathfrak{A}$ onto a C^*-algebra $\mathfrak{A}'$ is a C^*-isomorphism followed by left multiplication by the unitary operator $\rho(I)$.

The paper concludes with a classification of extreme points for factors.

Reviewer: Edwin Hewitt

For a scan of this review see the web version.

MSC:

46L05 General theory of C^*-algebras

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