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Quasigroups. I. (English) Zbl 0063.00039

Trans. Am. Math. Soc. 54, 507-519 (1943).

Introduction: A theory of non-associative algebras has been developed without any assumption of a substitute for the associative law, and the basic structure properties of such algebras have been shown to depend upon the possession of almost these same properties by related associative algebras.

It seems natural then to attempt to obtain an analogous treatment of quasigroups. We shall present the results here. Most of the results in the literature on quasigroups do depend upon special associativity conditions but no assumption of such conditions is necessary for our theorems.

Every quasigroup $\mathfrak{G}$ may be associated with the group $\mathfrak{G}$, of nonsingular transformations generated by its multiplications. The isotopy of two quasigroups may then be defined and, as in the case of algebras, two groups (that is associative quasigroups) are isotopic if and only if they are isomorphic. Every quasigroup is isotopic to a loop, that is, a quasigroup with an identity element, and we derive all further results for loops.

The concepts of coset and normal divisor may be defined for loops without any assumption of associativity. Then a subloop $\mathfrak{H}$ of $\mathfrak{G}$ is a normal divisor of $\mathfrak{G}$ if and only if $\mathfrak{H} = e\Gamma$ where e is the identity of $\mathfrak{G}$ and Γ is a normal divisor of the group $\mathfrak{G}$. Isotopic loops have corresponding normal divisors, and loops which are isotopic to simple loops are simple. We also combine the concepts of isotopy and homomorphism to yield a new concept of homotopy of loops. Then we show that if a loop $\mathfrak{G}$ is homotopic to a loop $\mathfrak{G}'$ it is homomorphic to a loop which is isotopic to $\mathfrak{G}'$.

Reviewer: Reviewer (Berlin)

MSC:

20N05	Loops, quasigroups
20N02	Sets with a single binary operation (groupoids)

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