Erdős, Pál

On the growth of the cyclotomic polynomial in the interval (0,1). (English) Zbl 0081.01703

Proc. Glasg. Math. Assoc. 3, 102-104 (1957).

Suppose n is a positive integer greater than unity and $F_n(x)$ is the n-th cyclotomic polynomial. Let A_n be the largest absolute value of any coefficient of $F_n(x)$, let B_n be the maximum value taken on by $F_n(x)$ on the interval $[0,1]$, and let C_n be the maximum value taken on by $F_n(x)$ on the disc $|x| \leq 1$. In a previous paper (Zbl 0038.01004) the author has shown that there is a positive constant c such that

$$C_n > \exp\exp\{c\log n/\log\log n\}$$

for infinitely many values of n. Since $A_n < C_n \leq nA_n$, this is equivalent to the corresponding assertion for A_n.

In the present paper the author gives a simpler proof of the more specific assertion that

$$B_n > \exp\exp\{c\log n\log\log n\} \qquad (*)$$

for infinitely many values of n, where c is a suitably chosen positive number. The values of n considered are products of a large number of very nearly equal primes and for these values of n the author investigates $F_n(x)$ at a carefully chosen value of x slightly less than $1-n^{-1/2}$. (Since $F_n(0)=F_n(1)=1$ if n has more than one prime factor, the maximum value of $F_n(x)$ on $[0,1]$ occurs at an interior point of the interval.) The argument requires only elementary results on the distribution of prime numbers. Although the author does not calculate c explicitly, his proof will give (*) for any c less than $\frac{1}{4}\log 2$, and a slight modification of the argument will give (*) for any c less than $\frac{2}{7}\log 2$. The author believes that perhaps (*) holds for any c less than $\log 2$, but that the present method of proof is not strong enough to give such a result. On the other hand, this would be as far as one could go, since, as the reviewer has remarked (cf. Zbl 0035.31102), it is almost immediate that if $\varepsilon > 0$, then

$$B_n \leq C_n \leq nA_n < \exp\exp\{(1+\varepsilon)(\log 2)\log n/\log\log n\}$$

for all large n.

Reviewer: P.T.Bateman

For a scan of this review see the web version.

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