Federer, Herbert
Geometric measure theory. (English) ‖ Zbl 0176.00801 ‖
Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen. 153. Berlin-Heidelberg-New York: Springer-Verlag. xiv, 676 p. (1969).

A first impression, that this is not a book but a monument, may need correcting: the 700 page volume is best described as a trilogy with applications, where each of the three, highly condensed and highly polished, main parts might correspond (at the very least) to a semester's lectures, but deserves to be studied much longer, and where the applications may direct the work of a lifetime.

The first part, which assumes only familiarity with point sets, comprises an up-to-date terse version of Grassmann algebra followed by an introduction to measure and integration, which takes us right into contemporary research and unsolved problems; this very necessary foundation is already of the utmost value to a serious student of analysis.

The second part, starting about p. 200, studies the structure of sets of finite k-measure in n-space, and includes the author's extension of the Besicovitch theory; it ends with some properties of highly differentiable functions and varieties.

This in its turn is all groundwork for the third part, which starts after p. 340 with a long chapter on homological integration, presumably the principal purpose of the book. There are points of contact with *H. Whitney*'s pioneering "Geometric Integration Theory". Princeton: Princeton University Press (1957; Zbl 0083.28204), but the differences, both in the aims and in the techniques, make it abundantly clear that this portion of mathematics has not stood still for twelve years. Many of the researches reported on arose from collaboration with W. H. Fleming in studying normal and integral currents, and they lead to general forms of the Gauss-Green theorem. A related topic is that of distributions whose first derivatives are vector-measures and comprises an account of researches originating also with de Giorgi, Krickeberg, Pauc, Goffman and others.

Finally the applications refer principally to the Problem of Plateau and to the regularity of solutions for certain partial differential equations, of interest in the calculus of variations. This final chapter is in the spirit of the very general definitions of Reifenberg, and it is also strongly influenced by the work of Almgren on a slightly modified form of the reviewer's generalized varieties.

The bibliography lists more than 250 books and papers by more than 170 authors.

Reviewer: L. C. Young (Madison)

For a scan of this review see the web version.

MSC:

28-02	Research exposition (monographs, survey articles) pertaining to measure and integration	Cited in **29** Reviews Cited in **2489** Documents

Keywords:

Grassmann algebra; measure and integration; geometric measure theory; sets of finite k-measure in n-space; Besicovitch theory; homological integration; normal and integral currents; Gauss-Green theorem; distributions; Problem of Plateau