Malgrange, Bernard

Ideals of differentiable functions. (English) Zbl 0177.17902

Studies in Mathematics. Tata Institute of Fundamental Research 3. London: Oxford University Press. 106 p. (1966).

Contemporary development of the local theory of differentiable functions in several variables can be roughly divided into two periods: The first period (1920–1961), with emphasis on analysis and differential topology, is centred on Whitney's achievements, and also on the works of M. Morse, Lojasiewicz, Thom, Malgrange, etc. A new step is provided by the introduction of local algebra under the influence of Malgrange, Mather and Tougeron, etc.

This important book is on the borderline between these two periods. It describes the most important results in analysis (with the exception of Thom's transversality theorem) in an elegant form. It provides the best available introduction to a modern attempt at algebrisation.

The first two chapters are devoted to Whitney's work. Particularly simple and elegant is the proof of spectral synthesis, in contrast to the complexity of the original version. This part also includes a "correct" proof of Sard's theorem. Chapter 3 introduces the local algebra which is needed for an abstract formulation of Weierstrass's Vorbereitungssatz in a general setting. The proof of Malgrange's preparation theorem is then presented, together with the Łojasiewicz-Hörmander division theorem, and Malgrange's generalisation to ideals generated by several analytic functions. A final chapter illustrates the duality between ideal theory and distribution theory, with applications to partial differential equations. -

The style of the author is extremely concise. Everything that is essential is said, with great precision. But, one feels that much more explanation would be helpful to the reader. For instance, it seems difficult to work on the algebraic aspects without a serious background in local algebra. For this reason, it is a pity that this book is not completed by a stock of examples and illustrative exercises which would help the student in his task.

It must be emphasized that the most important theorems presented here are not only theoretical results, but also technical tools that can be used to solve various types of problems. The author presents some examples, not enough for such a subject however, of the way Malgrange's preparation theorem can be used; but a similar methodology should be devoted to almost every other "big theorem" in this book.

Always looking for the best formal version of the subject, the author has failed to include in his book some ingenious tricks he had already used during the discovery of his theorems. At that time, his starting point was a division of a C^∞ function by a polynomial with indeterminate coefficients and he used then a substitution to get the final result. In his book, the simplicity of this elementary argument does not appear in the same illuminating fashion. As far as formal exposition is concerned, this does not matter; but the reader will certainly lose something of the heuristic path followed by a mathematician during his research.

Some slight misprints are likely to disturb the reader; for instance, on pages 22 and 24 you should read $F \in \hat{M}$ (and not $F \in M$). On page 35 the same letter f is used twice with different meanings so that $f \in O_m$ and $f \in O_n$! But never mind! A fundamental book indeed!

Reviewer: Georg Glaeser (Strasbourg)

For a scan of this review see the web version.

MSC:

46E25	Rings and algebras of continuous, differentiable or analytic functions	
46-02	Research exposition (monographs, survey articles) pertaining to functional analysis	

Keywords:

ideals of differentiable functions; local algebra; duality between ideal theory and distribution theory;

applications to partial differential equations