Varlet, J.

Ordered algebraic structures. (Structures algébriques ordonnees.) (French) Zbl 0358.06010

Liège: Universite de Liège, Institut de Mathematique. 134 p. (1975).

This book presents the materical of a university course on lattices, semilattices and tournaments. It is good, however, not only for students who want to look for a general orientation within these theories but also for those who regard its study as a first step towards independent researches. For instance, some recent results are treated or mentioned, too, and several unsolved problems are raised. The subject-matter is divided into nine chapters. Chapter 1 introduces the basic concepts concerning partially ordered sets, semilattices and lattices and deals with some elementary properties of complete lattices. In Chapter 2, distributive and modular lattices and semilattices are discussed. A semilattice is called distributive if to any elements x, y, z with $x \cap y \leq z$ there can be found $x_1$and y_1 such that $x \leq x_1$, $y \leq y_1$ and $x_1 \cap y_1 = z$; it is called modular if the same is true under the additional condition $z \leq y$ and with $y_1 = y$. It is pointed out that a subsemilattice or a homomorphic image of a distributive (or a modular) semilattice is not necessarily of the same kind. Chapter 3 discusses the diverse forms of complementation and investigates which of them are homomorphically invariant. Moreover, several characterizations of Boolean lattices are given. Chapter 4 presents the theory of ideals and filters in semilattices as well as the representation of distributive lattices as set rings and as join spaces (in the sense of *W. Prenowitz* and *J. Jantosciak* [J. Reine Angew. Math. 257, 100–128 (1972; Zbl 0264.50002)]). A remarkable concept introduced here is the a-distributivity but we call the attention of the reader to the fact that 0-distributivity of a lattice with 0 does not imply 0-modularity defined in Chapter 3. Chapter 5 treats the closure operations and their applications in partially ordered sets. A closure operation φ of an ordered groupoid $(G; \cdot)$ is called multiplicative if $\varphi(ab) = \varphi(a)\varphi(b)$ for all $a, b \in G$; there are given several theorems characterizing them. The family $\Phi(P)$ of closure operations on a partially ordered set P can be ordered in a natural way. Conditions are given for $\Phi(P)$ to form a lattice and in such cases the properties of this closure lattice are investigated. As a more important application, MacNeille's completion theorem is proved. Chapter 6 begins with the basic properties of congruences in algebras, then standard theorems on congruences of lattices can be read. They are followed by some interesting results on the congruences of semilattices, for instance by conditions assuring the congruence lattice to be distributive or Boolean. In Chapters 7–8 pseudo-complemented and relatively pseudo-complemented semilattices and lattices as well as their congruences are treated. In particular, Stone lattices are characterized. Chapter 9 deals profoundly with the lattice of all convex subsets of tournaments; the most interesting result is a theorem of Helly type. It is shown that the congruences of a tournament form a pseudo-complemented distributive lattice and conditions are given for any two congruences to be permutable.

Reviewer: G. Szász

For a scan of this review see the web version.

MSC:

06-01	Introductory exposition (textbooks, tutorial papers, etc.) pertaining to ordered structures
06A06	Partial orders, general
06A12	Semilattices
06A15	Galois correspondences, closure operators (in relation to ordered sets)
06B05	Structure theory of lattices
06B10	Lattice ideals, congruence relations
06C05	Modular lattices, Desarguesian lattices
06D05	Structure and representation theory of distributive lattices
06D15	Pseudocomplemented lattices
06F05	Ordered semigroups and monoids