Erdős, Paul; **Sárkőzy, András**

On differences and sums of integers. I. (English) Zbl 0404.10029

J. Number Theory 10, 430-450 (1978).

A set $B=\{b_1,b_2,\dots,b_i\}\subset\{1,2,\dots,N\}$ is a difference intersector set if for any set $A=\{a_1,a_2,\dots,a_j\}\subset\{1,2,\dots,N\}$, $j=\varepsilon N$ the equation $a_x-a_y=b$ has a solution. The notion of a sum intersector set is defined similary. Using exponential sum techniques, the authors prove two theorems which in essence imply that a set which is well-distributed within and amongst all residue classes of small modules is both a difference and a sum intersector set. The regularity of the distribution of the non-zero quadratic residues (mod p) allows the theorems to be used to investigate the solubility of the equations $\left(\frac{a_x-a_y}{p}\right)=+1$, $\left(\frac{a_r-a_s}{p}\right)=-1$, $\left(\frac{a_t-a_u}{p}\right)=+1$, and $\left(\frac{a_v-a_w}{p}\right)=-1$. The theorems are also used to establish that "almost all" sequences form both difference and sum intersector sets.

Reviewer: M.M.Dodson

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MSC:

11B83	Special sequences and polynomials
11B13	Additive bases, including sumsets
11P99	Additive number theory; partitions
11D85	Representation problems
11L03	Trigonometric and exponential sums, general

Keywords:

difference intersector set; sum intersector set; distribution quadratic residues; sequence of integers

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