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The small dispersion limit of the Korteweg-de Vries equation. I. (English) Zbl 0532.35067

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In a series of three papers, the authors have analyzed the bahavior of solutions $u(x,t;\epsilon)$ of the equation $u_t - 6uu_x + \epsilon^2 u_{xxx} = 0$ as $\epsilon \to 0$ while the initial values are fixed. Only nonpositive initial data were considered; in that case the limiting reflection coefficient vanishes. It is known from computer studies that for t greater than a critical time, independent of ϵ, dependent only on the initial data, $u(x,t;\epsilon)$ becomes oscillatory as $\epsilon \to 0$. The wavelength of these oscillations is of the order $0(\epsilon)$, and their amplitude is independent of ϵ. This indicates that $\lim u(x,t;\epsilon \to 0)$ exists only in the weak sense. This paper represents the first part of the work. The paper was organized as follows: In Section 1, the direct scattering problem for given initial data, assumed for simplicity to have a single local minimum, was solved asymptotically. In Section 2, the Kay-Moses explicit solution of the reflectionless inverse problem was used to carry out the limit $\epsilon \to 0$. The authors have shown that $\bar{u}(x,t) = \lim u(x,t;\epsilon \to 0)$ exists in the sense of weak convergence in $L_2(R)$ with respect to x, and that the weak limit $\bar{u}$ can be described as $\bar{u} = \partial_{xx} Q^*$. The function $Q^*(x,t)$ is determined by solving a quadratic programming problem

$$Q^*(x,t) = \min_{0 \le \psi \le \phi} Q(\psi; x,t).$$

Here $Q(\psi; x,t)$ is a quadratic functional of ψ, which depends linearly on the parameters x and t, while the function ϕ is determined by the initial data. In Section 3 the authors have shown that Q is continuous in a weak sequential topology, and that the space of admissible functions is compact in that topology. They further showed that Q is a strictly convex function; since the admissible functions form a convex set, this implies not only that the minimum of Q is taken on at a unique function, but that this function is the only one which satisfies variational conditions. The variational conditions were then converted to a Riemann-Hilbert problem.

Reviewer: L.-Y.Shih

MSC:

35Q99 Partial differential equations of mathematical physics and other areas of application
35B40 Asymptotic behavior of solutions to PDEs
47A40 Scattering theory of linear operators
35P25 Scattering theory for PDEs
35A15 Variational methods applied to PDEs

Keywords:

KdV equation; nonpositive initial data; weak dispersion limit; scattering transform method; minimum problem

Full Text: DOI

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