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Unique continuation for Schrödinger operators in dimension three or less. (English)

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We show that the differential inequality $|\Delta u| \leq v|u|$ has the unique continuation property relative to the Sobolev space $H^{2,1}_{loc}(\Omega)$, $\Omega \subset R^n$, $n \leq 3$, if v satisfies the condition

$$(K_n^{loc}) \quad \lim_{r\to 0} \sup_{x\in K} \int_{|x-y|<r} |x-y|^{2-n} v(y)dy = 0$$

for all compact $K \subset \Omega$, where if $n = 2$, we replace $|x-y|^{2-n}$ by -log$|x-y|$. This resolves a conjecture of B. Simon on unique continuation for Schrödinger operators, $H = -\Delta + v$, in the case $n \leq 3$. The proof uses Carleman's approach together with the following pointwise inequality valid for all $N = 0, 1, 2, \ldots$ and any $u \in H_c^{2,1}(R^3 - \{0\})$;

$$\frac{|u(x)|}{|x|^N} \leq C \int_{R^3} |x-y|^{-1} \frac{|\Delta u(y)|}{|y|^N} dy$$

for a.e. x in R^3.

MSC:

35B60 Continuation and prolongation of solutions to PDEs
35J10 Schrödinger operator, Schrödinger equation
35R45 Partial differential inequalities and systems of partial differential inequalities

Keywords:

unique continuation; Schrödinger operators; Sobolev space

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