

Hackbusch, Wolfgang

On the regularity of difference schemes. II: Regularity estimates for linear and nonlinear problems. (English) [Zbl 0535.65071](#)

Ark. Mat. 21, 3-28 (1983).

[For the first part see *ibid.* 19, 71-95 (1981; [Zbl 0462.65058](#)).]

Let a boundary value problem for a second order elliptic equation be written in the form $Lu = f$ and let $L_h u_h = f_h$ be its discrete analogue. The author analyzes the important problem of getting a priori estimates for L_h^{-1} which can be regarded as discrete analogue of the coercitivity estimates

$$(1) \quad \|L^{-1}\|_{H^s \rightarrow H^{s+2}} \leq C$$

with H^s being either the Sobolev space W_2^s or the Hölder space C^s . He gives an interesting theorem which connects the s_1 -regularity of the discrete problems with their s_0 -regularity ($s_0 < s_1$) and estimates (1) for the differential case with $s \in [s_0, s_1]$. This theorem enables him to get strong results about the s -regularity of a few difference schemes for the Poisson equation in a general region $\Omega \subset R^2$ with a smooth boundary. Some generalizations for nonlinear problems are given.

Reviewer: [E.D'jakonov](#)

MSC:

- [65N15](#) Error bounds for boundary value problems involving PDEs
- [65N12](#) Stability and convergence of numerical methods for boundary value problems involving PDEs
- [35J05](#) Laplace operator, Helmholtz equation (reduced wave equation), Poisson equation
- [35J25](#) Boundary value problems for second-order elliptic equations
- [35J65](#) Nonlinear boundary value problems for linear elliptic equations

Cited in **7** Documents

Keywords:

[regularity estimates](#); [a priori estimates](#); [coercitivity estimates](#); [Poisson equation](#)

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