

**Flum, J.**

**Modelltheorie - topologische Modelltheorie.** (English) Zbl 0538.03033  
Jahresber. Dtsch. Math.-Ver. 86, 69-82 (1984).

This pleasant survey consists of two parts; classical and topological model theory. In the first part, the author takes up a few topics that are of interest to mathematics in general. So he discusses the categoricity in power of the algebraically closed fields, the classification of Abelian groups with respect to elementary equivalence, and significance of existentially closed structures and of non-standard analysis. Purely model theoretic topics like stability are not mentioned. Some people might have liked to see ultraproducts and the solution of a conjecture of Artin in this part. In the second part, invariant sentences are introduced as a good vehicle for the topological model theory. A monadic second order sentence  $\phi$  is said to be invariant if for each topological structure  $(\mathfrak{A}, \sigma)$  and each open basis  $\sigma_0 \subseteq \sigma$ ,  $(\mathfrak{A}, \sigma) \models \phi$  iff  $(\mathfrak{A}, \sigma_0) \models \phi$ . A syntactical characterization of such sentences is given and the Löwenheim-Skolem-Tarski theorem is shown to hold. And then, the author skilfully exploits various previous work on order topologies, ordered groups, and fields with valuations to produce interesting results. For instance, the invariant theory of  $T_3$  spaces is decidable, and the invariant equivalence of such spaces is reduced to a number of points of various topological types (like accumulation points). He shows the approximation theorem for many V- topologies on a field from that of valuations. This part contains a small number of sample results that are in the monograph that the author wrote with *M. Ziegler* [Topological model theory (1980; [Zbl 0421.03024](#))], and the author does not go much into other approaches to the topological model theory like the study of lattices of closed sets and the use of new quantifiers. (For these, cf. the papers by *C. W. Henson*, *C. G. Jockusch jun.*, *L. A. Rubel*, and *G. Takeuti* [Diss. Math. 143 (1977; [Zbl 0399.03019](#))] and by *J. Sgro* [Ann. Math. Logic 11, 173- 193 (1977; [Zbl 0387.03010](#))] respectively, for instance.)

Reviewer: [M. Yasuhara](#)

**MSC:**

- [03C99](#) Model theory
- [03C85](#) Second- and higher-order model theory
- [03-02](#) Research exposition (monographs, survey articles) pertaining to mathematical logic and foundations

**Keywords:**

classical model theory; survey; topological model theory; invariant sentences; order topologies