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Completeness of states of the generalized Heisenberg magnet. (Russian. English summary)

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Zap. Nauchn. Semin. Leningr. Otd. Mat. Inst. Steklova 134, 169-189 (1984).

The author proves the completeness of the multiplet systems based on Bethe vectors for the general Heisenberg magnet model. The method is based on the connection between the algebraic model of Bethe and the representation theory of the Lie group $\mathrm{SU}(p+1)$. Firstly, the basic results on the general Heisenberg magnet are given. The algebraic system of equations describing a Heisenberg magnet is studied for the case $N \to \infty$, where N is the number of all possible states of the magnet. The solutions of this system are parametrized by Bethe vectors. The number of these vectors is calculated and the hypothesis of completeness is rigorously formulated. Secondly, a fundamental combinatorial formula is proven, based on the following relation between the characters ω_k^m of $\mathrm{SU}(p+1)$: $\omega_k^m \otimes \omega_k^m = \omega_k^{m-1} \otimes \omega_k^{m+1} + \omega_{k-1}^m \otimes \omega_{k+1}^m$. This formula is used then to prove the completeness for the general Heisenberg magnet. The completeness hypothesis results from the equality of the number $Z(N, s|\mu)$ of Bethe vectors and the multiplicity e_μ of the irreducible representation V_μ of $\mathrm{SU}(p+1)$, which signature μ, in the tensorial product $\otimes_n W_n = \Sigma e_\mu V_\mu$ of representations. Finally, a method for obtaining the irreducible representations V_η in the tensorial product $V_\lambda \otimes V_\mu$ of irreducible representations is given.

Reviewer: G. Zet

MSC:

53D50	Geometric quantization
82D20	Statistical mechanical studies of solids
22E70	Applications of Lie groups to the sciences; explicit representations

Keywords:

Bethe model; character; irreducible representations

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