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Right ideals in a right distributive groupoid. (English) Zbl 0544.20052

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Let G denote a groupoid, $<G>$ denotes the groupoid generated by G under set product [e.g. $GG^2 \in <G>$ where $GG^2 = \{a(bc)| \quad a,b,c \in G\}$], R(G) denotes the set of right ideals of G, and $P(G) = \{G^n|$ n is a positive integer$\}$ where $x^{n+1} = x^n x$ for $x \in G$. It is well known that if G is a semigroup then: (i) R(G) is a semigroup under set product, (ii) $<G> \subseteq R(G)$, (iii) $<G>$ is totally ordered under inclusion. In general (i), (ii), (iii) are not true. However, in this paper, it is shown that if G is a right distributive groupoid [i.e. $(xy)z = (xz)(yz)$] then R(G) is a right distributive groupoid and conditions (ii) and (iii) are satisfied. Examples indicate that, in general, $P(G) \neq <G>$ and that $<G>$ is right distributive does not imply G is right distributive (although the converse is true). The following are equivalent: (a) $<G>$ is right distributive; (b) if $Y, V \in <G>$ such that $Y \neq G$ then $YV = YG$ and $(GV)G = G^3$; if $A, B, C \in <G>$, then $(AB)C = (AG)G$. Finally if $<G>$ is right distributive the following conditions, on $<G>$, are characterized: $<G> = P(G)$, commutativity, associativity, distributivity (both sides).

MSC:

20M10 General structure theory for semigroups
20M12 Ideal theory for semigroups

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