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Espaces préhomogènes de type parabolique. (Thèse d'Etat). (English) Zbl 0546.22019

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The theme of this thesis is the study of prehomogeneous vector spaces of parabolic type. It is divided into two parts, one is the algebraic study and is devoted mainly to the definition and the classification of prehomogeneous vector spaces of parabolic type, and the other is the analytic study which is applied to the research on local zeta functions associated with them.

In the algebraic study, the author found a natural structure of prehomogeneous vector space in a simple Lie algebra. That is; let $\mathfrak{g}$ be a simple Lie algebra, $\mathfrak{h}$ a Cartan subalgebra of $\mathfrak{g}$, and R the root system with respect to $(\mathfrak{g}, h)$. We fix Ψ a base of R. Let θ be a subset of Ψ. We put $d_p(\theta) = \{X_\theta \in \mathfrak{g}; [H, X] = 2pX\}$ for $p \in Z$. Here, H^θ is the element of $\mathfrak{h}_\theta$ defined by $\alpha(H_\theta) = 2$ for $\alpha \in \Psi - \theta$ and $\alpha(H_\theta) = 0$ for $\alpha \in \theta$. In particular, we denote $\ell_\theta = d_0(\theta)$. Since $\{d_p(\theta)\}_{p\in Z}$ gives a gradation of $\mathfrak{g}$, i.e., $[d_i(\theta), d_j(\theta)] \subset d_{i+j}(\theta)$. The Lie algebra ℓ_θ acts on $d_p(\theta)$, i.e., $[\ell_\theta, d_p(\theta)] \subset d_p(\theta)$ and this action causes a representation of the Lie group L_θ in $GL(d_p(\theta))$ whose Lie algebra is ℓ_θ. Then, $(L_\theta, d_p(\theta))$ is a prehomogeneous vector space.

In particular, when $p = 1$, the pair $(L_\theta, d_1(\theta))$ is called a prehomogeneous vector space of parabolic type and furthermore, when $Card(\psi - \theta) = 1$, it is proved that $(L_\theta, d_1(\theta))$ is an irreducible prehomogeneous vector space. The author decides when such prehomogeneous vector space of parabolic type is regular and classifies all of them when $\mathfrak{g}$ is simple. It is proved that "almost all" reduced regular irreducible prehomogeneous vector spaces are obtained as a prehomogeneous vector space of parabolic type. The author determines the real forms of the prehomogeneous vector spaces of parabolic type by making use of the real forms of Dynkin diagrams (Chapter 2), carries out the classification of regular prehomogeneous vector spaces of parabolic type which are Q-irreducible (Chapter 3) and proves that the ring of regular invariant functions on $X_\theta = G \cdot (\mathfrak{h}_\theta + \sum_{p\geq 1} d_p(\theta))$ is integrally closed, which is an affirmative answer for Borho's conjecture in a certain case.

In the analytic study, the author gives a relation between local zeta functions on prehomogeneous vector spaces and intertwining operators in some cases, and studies on the poles of local zeta functions.

Reviewer: M.Muro

MSC:

22E46	Semisimple Lie groups and their representations
22E60	Lie algebras of Lie groups
14M17	Homogeneous spaces and generalizations
17B05	Structure theory for Lie algebras and superalgebras
17B70	Graded Lie (super)algebras

Keywords:

prehomogeneous vector spaces of parabolic type; algebraic study; analytic study; local zeta functions; simple Lie algebra; classification of regular prehomogeneous vector spaces; intertwining operators; poles