Lorch, Lee; **Newman, Donald J.**

On the composition of completely monotonic functions and completely monotonic sequences and related questions. (English) Zbl 0547.26010

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The authors answer several previously open questions about c.m. (completely monotonic) sequences and functions. (1) If W(x) is c.m. on $[a,\infty)$ and $\{\Delta x_k\}$ is c.m. with $x_0\geq a$, then $\{W(x_k)\}_0^\infty$ is c.m. Also, the sequence $\{\mu_k^\lambda\}, \quad \mu_0=1, \quad \mu_k>0, \quad k=1,2,...,$ is c.m. for all $\lambda>0$ if and only if $\mu_k=\exp(-\nu_k)$ with $\{\Delta\nu_k\}^c$.m., $\nu_0=0$. (2) If V'(x) is c.m. on $(0,\infty)$, and $\{\Delta x_k\}_0^\infty$ is c.m., then $\{\Delta V(x_k)\}_0^\infty$ is c.m. (3) Partial converse of (1): Let $W(x)>0 \quad (0\leq x<\infty), \quad W'(x)<0 \quad (0<x<\infty)$, and let $W'(0^+)$ existe (finite). If $\{W(\lambda x_k)\}$ is c.m. for all small $\lambda>0$ and $x_0\geq 0$, then $\{\Delta x_k\}$ is c.m. However, (4) if f is c.m. on $[0,\infty)$ there is a ϕ (t) with $\phi(0)=0, \quad f(\phi(t))$ c.m. on $[0,\infty)$, but ϕ '(t) not c.m. on $(0,\infty)$. (5) If $\phi(f(x))$ is c.m. on $(0,\infty)$ for all f(x) that are c.m. on $(0,\infty)$ then ϕ (x) is absolutely monotonic on (0,∞). (6) If f and g are convex and nonnegative and $k\geq 1$ then $(f^k+g^k)^{1/k}$ is convex. (7) If f is monotonic of order $N\geq 2$ then $[f(x)]^{1/(N-1)}$ is convex. (8) If f is monotonic of order N and $\lambda>1$, then $[f(x)]^\lambda$ is monotonic of order N if $N=1,2,3 \quad or \quad 4$ but not necessarily if $N\geq 5$. (9) If f is c.m. and $\lambda>1$ then $[f(t)]^\lambda$ is monotonic of order 5. Many interesting special cases and corollaries are also given.

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