Silverman, B. W.

Spline smoothing: The equivalent variable kernel method. (English) Zbl 0547.62024

Ann. Stat. 12, 898-916 (1984).

The cubic spline estimator of the regression curve is related with the existence of a weight function which enables to perform a nonparametric estimation of the function. The relation between this method and the kernel approach is fixed under suitable conditions with the aims of giving an intuitive insight into spline-smoothing methods.

The main result is that for a sequence of probability distribution functions F_n, the weight function corresponding to a design point is similar to a specific kernel function k if n is sufficiently large, the smoothing parameter λ is small and the design point t is not near the boundaries of the interval (a,b) on which the sequence is defined. k is centred in t with bandwidth $(\lambda/F'(t))^{1/4}$, where $F = \lim_{n\to\infty} F_n$ is an absolutely continuous distribution function on (a,b).

This result is given in theorem A under assumptions related with the existence of F, the boundness of its first and second derivatives and restrictions on how fast λ should tend to zero. From three lemmas and two propositions given in Section 4 the theorem is obtained. As the approximation of the weight function is not good when t is close to the boundaries of (a,b) a solution is stated in theorem B of Section 5.

Illustrations of the performance of the approximation of the weight functions are given. Some applications are derived in connection with the hat matrix and the estimation of a density function.

Reviewer: C.N.Bouza

MSC:

62G05	Nonparametric estimation
65D10	Numerical smoothing, curve fitting
62J02	General nonlinear regression
65C99	Probabilistic methods, stochastic differential equations
46E35	Sobolev spaces and other spaces of "smooth" functions, embedding theorems, trace theorems

Keywords:

variable kernel; roughness penalty; penalized maximum likelihood; curve estimation; density estimation; Sobolev spaces; cubic spline estimator; kernel approach; spline-smoothing methods; approximation of the weight functions; hat matrix

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