Fujimoto, Takao; Herrero, Carmen; Villar, Antonio

A sensitivity analysis for linear systems involving M-matrices and its application to the Leontief model. (English) Zbl 0556.15003

Linear Algebra Appl. 64, 85-91 (1985).

Let M^0 and M^* be (n×n) non-singular M-matrices (n≥ 2). (That is: M^0 and M^* may be written as $M^0 = \lambda I - A^0$, $M^* = \lambda I - A^*$ where λ is a positive scalar, and A^0 and A^* are non-negative matrices, and $(M^0)^{-1}$, $(M^*)^{-1}$ are also non-negative.) M^0 and M^* may differ only in the first s ($0 < s < n$) columns. Let w^0 and w^* be strictly positive n-vectors, which coincide in their last (n-s) entries. Let $p^0 = (M^0)^{-1}w^0$, $p^* = (M^*)^{-1}w^*$. Then if $(p^0M^*)_i > w_i^*$ for $i \in S$, the authors prove $\min_{i\in S}\{p_i^*/p_i^0\} < \min_{i\in R}\{p_i^*/p_i^0\}$, where $S = \{1, 2, ..., s\}$, $R = \{s + 1, ..., n\}$. This is a partial generalization of Theorem 21 of *G. Sierksma* [Linear Algebra Appl. 26, 175-201 (1979; Zbl 0409.90027)]; see also the reviewer's paper [Non-negative matrices and Markov chains (1981; Zbl 0471.60001 pp. 35- 39], in that changes in the M-matrix { from M^0 to M^*} are also permitted.

Reviewer: Eugene Seneta (Sydney)

MSC:

15A06 Linear equations (linear algebraic aspects)
93B35 Sensitivity (robustness)
15B48 Positive matrices and their generalizations; cones of matrices
91B60 Trade models

Keywords:

sensitivity analysis; M-matrices; Metzler theorem; Morishima theorem; Leontief model

Full Text: DOI

References:

[1] Berman, A.; Plemmons, R.J., Nonnegative matrices in the mathematical sciences, (1979), Academic New York · Zbl 0484.15016
[2] Debreu, G.; Herstein, I.N., Nonnegative square matrices, Econometrica, 21, 567-607, (1953) · Zbl 0051.00901
[3] Elsner, L.; Johnson, C.R.; Neumann, M., On the effect of the perturbation of a nonnegative matrix on its Perron eigenvector, Czechoslovak math. J., 32, 99-109, (1982) · Zbl 0501.15010
[4] Fiedler, M.; Pták, V., On matrices with non-positive off-diagonal elements and positive principal minors, Czechoslovak math. J., 12, 382-400, (1962) · Zbl 0131.24806
[5] Fujimoto, T., An elementary proof of Okishio's theorem for models with fixed capital and heterogeneous labour, Metroeconomica, 33, 21-27, (1981)
[6] T. Fujimoto, C. Herrero, and A. Villar, Technical changes and their effects on the price structure, \textit{Metroeconomica}, to appear.
[7] Metzler, L.A., A multiple-country theory of incomes transfers, J. political economy, 59, 14-29, (1951)
[8] Metzler, L.A., Taxes and subsidies in Leontief's input-output model, Quart. J. econom., 65, 433-438, (1951)
[9] Morishima, M., Equilibrium, stability and growth, (1964), Clarendon Oxford · Zbl 0117.15406
[10] Murata, Y., Mathematics for stability and optimization of economic systems, (1977), Academic New York
[11] Seneta, E., Nonnegative matrices, (1973), Wiley New York · Zbl 0278.15011
[12] Seneta, E., Nonnegative matrices and Markov chains, (1981), Springer Berlin · Zbl 0471.60001
[13] Sierksma, G., Non-negative matrices: the open Leontief model, Linear algebra appl., 26, 175-201, (1979) · Zbl 0409.90027