Huisken, Gerhard

Flow by mean curvature of convex surfaces into spheres. (English) Zbl 0556.53001

J. Differ. Geom. 20, 237-266 (1984).

Let M_0 be a smooth closed convex hypersurface with everywhere positive curvatures in Euclidean space $\mathbb{R}^{n+1}$. Suppose that M_0 is smoothly deformed (that is, embedded into a smooth family (M_t, $t \geq 0$) of hypersurfaces depending on a time parameter) such that, at every time t, the rate of change of M_t in direction of the inward unit normal vector is equal to the (positive) mean curvature of the hypersurface at the point considered. The author shows that the parabolic evolution equation describing the problem has a smooth solution on a finite time interval $0 \leq t \leq T$, and the $M_t's$ converge to a point as $t \to T$. Moreover, if the surfaces undergo suitable homotheties and the time parameter is transformed appropriately into a parameter $\tilde{t}$, $0 \leq \tilde{t} < \infty$, it is shown that the normalized surfaces converge to a sphere in the C^∞-topology as $\tilde{t} \to \infty$. As the author says, his approach is inspired by a paper of *R. S. Hamilton* [J. Differ. Geom. 17, 255-306 (1982; Zbl 0504.53034)], and he can use many of the methods developed there. The case $n = 1$ was treated by *M. E. Gage* [Invent. Math. 76, 357-364 (1984; Zbl 0542.53004)].

{Reviewer's remark: A similar problem, related in spirit though not in the methods, was treated by *W. J. Firey* [Mathematika 21, 1-11 (1974; Zbl 0311.52003)]. There the rate of change of the support function is proportional to the Gauss curvature.}

Reviewer: R.Schneider

MSC:

53A07 Higher-dimensional and -codimensional surfaces in Euclidean and related n-spaces
35G10 Initial value problems for linear higher-order PDEs
35K25 Higher-order parabolic equations
52A20 Convex sets in n dimensions (including convex hypersurfaces)

Keywords:

convex hypersurface; mean curvature; evolution equation

Full Text: DOI