Todorčević, S.

Trees and linearly ordered sets. (English) [Zbl 0557.54021]

[For the entire collection see Zbl 0546.00022.]

The survey will be useful for everybody who is working in general topology or set theory. Many presented problems are of set theoretical nature and there is a large number of consistency results. The paper is divided into 9 sections and each ends with historical remarks. The reader will find exactly 99 results presented in the form of a theorem, lemma, corollary or a proposition. In order to have some understanding of nature and quality of the results I quote some of them: (1) If \( L \) is a linearly ordered set, then \(|L| \leq 2^{\omega_0(L)}\), where \( \omega_0(L) \) means the supremum of all cardinals of the form \(|A|\) and \( A \) is a well-ordered subset of \( L \) (Theorem 3.4; Hausdorff 1914, Urysohn 1924), (2) There is a Lindelöf \( \omega_1 \)-metrizable non-linearly orderable topological space (Corollary 4.5; Kurepa 1963), (3) There is an uncountable linearly ordered set \( L \) such that \( L^2 \) is the union of countably many chains (Theorem 5.3; Shelah 1976), (4) There is a family of 2 nonisomorphic rigid Aronszajn trees (Theorem 5.8; Avraham 1979, Baumgartner, the author 1980), (5) Assume \( \diamondsuit \). Then there is a homogeneous non-reversible Suslin continuum \( K \) such that \( K \times K \) has a dense metrizable subspace (Theorem 6.11; Jensen 1974), (6) If there is an inaccessible cardinal \( \kappa \), then there is a forcing notion \( P \) such that in \( V^P \) we have \( \kappa = \aleph_2 \) and there are no Kurepa trees (Theorem 8.12; Silver 1971), (7) Assume that both \( A \) and \( \omega_1 \setminus A \) are stationary subsets of \( \omega_1 \). Then \( Y(A) \) is a compact first countable Corson space with no dense metrizable subspace (Theorem 9.14; the author 1981).

Reviewer: W.Kulpa

MSC:
- 54F05 Linearly ordered topological spaces, generalized ordered spaces, and partially ordered spaces
- 54A35 Consistency and independence results in general topology
- 03E35 Consistency and independence results

Keywords:
- Lindelöf \( \omega_1 \)-metrizable non-linearly orderable topological space; space; rigid Aronszajn trees; homogeneous non-reversible Suslin continuum; dense metrizable subspace; compact first countable Corson space; Lindelöf \( \omega_1 \)-metrizable non-linearly orderable topological space