Duistermaat, J. J.

On the similarity between the Iwasawa projection and the diagonal part. (English)

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Mém. Soc. Math. Fr., Nouv. Sér. 15, 129-138 (1984).

Let G be a real connected semi-simple Lie group with finite center and $G=KAN$ be its Iwasawa decomposition. Let g be the Lie algebra of G and k be the Lie algebra of K. Let s denote the orthogonal complement of k in g with respect to the Killing form, (s always denoted by p), then the Cartan decomposition $G=K\cdot \exp s$ yields that $s\to G\to K\setminus G$ is a diffeomorphism from s onto the (non-compact Riemannian) symmetric space $K\setminus G$. The Iwasawa projection H from G onto the Lie algebra a of A is defined by $x\in K\cdot \exp H(x)\cdot N$, $x\in G$. Let γ be the mapping $H\cdot \exp :s\to a$, and let π be the orthogonal projection $s\to a$ with respect to the Killing form.

The main results of this paper is the following theorem. There is a real analytic map $\psi :s\to K$ such that $(i)\quad \phi_X:k\to k\cdot \psi(Adk^{-1}(X))$ is a diffeomorphism from K onto K, for each $X\in s$, $(ii)\quad \gamma(Ad\psi(X)^{-1}(X))=\pi(X)$ for all $X\in s$. - This means that the Iwasawa projection can be turned into the orthogonal projection π by the action $Ad\psi(X)^{-1}\in AdK$, and the element ψ (X) of K depends analytically on $X\in s$. Some results obtained by *J. J. Duistermaat*, *J. A. C. Kolk* and *V. S. Varadarajan* [Compos. Math. 49, 309-398 (1983; Zbl 0524.43008)] are used in the argument of the theorem of this paper. For $G=SL(2,\mathbb{R})$, an explicit consideration is also given.

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