Loday, Jean-Louis; **Quillen, Daniel**
Cyclic homology and the Lie algebra homology of matrices. (English) ⟦Zbl 0565.17006⟧
Comment. Math. Helv. 59, 565-591 (1984).

For associative algebras A with identity over commutative rings k A. Connes and B. L. Tsygan defined the cyclic homology $HC_i(A)$, $i \geq 0$. The paper approaches the subject starting from a double complex suggested by Tsygan's work. This technical tool is employed to simplify and develop the theory of Connes and Tsygan, and to give a complete proof to Theorem 6.2 announced by the authors and independently by Tsygan. There are constructed maps from cyclic homology to the de Rham cohomology, a product is introduced, and the notion of reduced cyclic homology is defined. The fifth section contains the computation of the cyclic homology for a free algebra (Proposition 5.4). For the case $k \subseteq \mathbb{Q}$ such computations were independently made by Tsygan. The main result in the last section is

Theorem 6.2: If $k \subseteq \mathbb{Q}$ then the cyclic homology is the primitive part of the homology of the Lie algebra of matrices. In particular, $HC_{*-1}(A)$ is an additive analogue of Quillen's K-functors.

Theorem 6.9 is a result on stability: for $i \leq n$, $H_i(\mathfrak{gl}_n(A), k)$ does not depend on n.

By Theorem 6.2 one can introduce on $HC_{*-1}(A)$ an increasing filtration F_* where the nth term consists of those primitive classes of the homology of $\mathfrak{gl}_\infty$ which are images of homology classes of $\mathfrak{gl}_n$. Conjecture 6.14 is not true in general. (It is true, nevertheless, for another, decreasing filtration.) For F_* this proposition (i.e. $F_n HC_{2n}(A) = 0$) only holds in the case when A is the coordinate ring of a smooth algebraic manifold. In general, $A = k[V]/(V)^2$, where $\dim V = \infty$, gives a counter-example (personal communication by B. L. Tsygan).

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MSC:

17B56   Cohomology of Lie (super)algebras
16E40   (Co)homology of rings and associative algebras (e.g., Hochschild, cyclic, dihedral, etc.)

Cited in **23** Reviews
Cited in **170** Documents

Keywords:

Hochschild homology; cyclic homology; Lie algebra of matrices; filtration; primitive classes

Full Text: DOI   EuDML