

**Fishburn, P. C.**

**Proportional transitivity in linear extensions of ordered sets.** (English) [Zbl 0566.06002](#)  
*J. Comb. Theory, Ser. B* 41, 48-60 (1986).

Let  $p_{ij}$  denote the proportion of all linear extensions  $> *$  of a partial order on  $\{1, 2, 3, \dots, n\}$  in which  $i > *j$ . The deterministic transitivity law for the subset  $\{1, 2, 3\}$  says that  $(p_{12} = 1, p_{23} = 1) \Rightarrow p_{13} = 1$ , and similarly for permutations of 123. The corresponding probabilistic or proportional transitivity law asserts that, for all  $(\lambda, \mu)$  in the unit square,  $(p_{12} \geq \lambda, p_{23} \geq \mu) \Rightarrow p_{13} \geq f(\lambda, \mu)$ , where  $f(\lambda, \mu)$  is the infimum of  $p_{13}$  over all finite posets that have  $p_{12} \geq \lambda$  and  $p_{23} \geq \mu$ .

It is shown that  $f(\lambda, \mu) = 0$  when  $\lambda + \mu < 1$ , and  $f(1, \mu) = \mu$ . Moreover,  $f(\lambda, 1-\lambda) \leq 1/e$ , and, when  $\lambda + \mu > 1$  and  $\max\{\lambda, \mu\} < 1$ ,  $f(\lambda, \mu) \leq 1 - (1 - \lambda)(1 - \mu)[1 - \log(1 - \lambda)(1 - \mu)]$ . The exact value of  $f$  is presently unknown for every case in which  $\lambda + \mu \geq 1$  and  $\max\{\lambda, \mu\} < 1$ .

**MSC:**

[06A06](#) Partial orders, general

Cited in **8** Documents

**Keywords:**

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**References:**

- [1] Fishburn, P.C, On the family of linear extensions of a partial order, *J. combin. theory*, 17, 240-243, (1974) · [Zbl 0274.06003](#)
- [2] Fishburn, P.C, On linear extension majority graphs of partial orders, *J. combin. theory*, 21, 65-70, (1976) · [Zbl 0294.06001](#)
- [3] Fishburn, P.C, A correlational inequality for linear extensions of a poset, *Order*, 1, 127-137, (1984) · [Zbl 0562.06002](#)
- [4] Lang, S, ()
- [5] Shepp, L.A, The  $\{XYZ\}$  conjecture and the FKG inequality, *Ann. probab.*, 10, 824-827, (1982) · [Zbl 0484.60010](#)
- [6] Stanley, R.P, Two combinatorial applications of the Alexandrov-Fenchel inequalities, *J. combin. theory ser. A*, 31, 56-65, (1981) · [Zbl 0484.05012](#)

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