**Fishburn, P. C.**

**Proportional transitivity in linear extensions of ordered sets.** (English) Zbl 0566.06002

J. Comb. Theory, Ser. B 41, 48-60 (1986).

Let $p_{ij}$ denote the proportion of all linear extensions $>*$ of a partial order on { 1,2,3,...,n} in which $i>*j$. The deterministic transitivity law for the subset { 1,2,3} says that $(p_{12}=1,p_{23}=1)\Rightarrow p_{13}=1$, and similarly for permutations of 123. The corresponding probabilistic or proportional transitivity law asserts that, for all ($\lambda$,$\mu$) in the unit square, $(p_{12}\geq \lambda,p_{23}\geq \mu)\Rightarrow p_{13}\geq f(\lambda,\mu)$, where f($\lambda$,$\mu$) is the infimum of $p_{13}$ over all finite posets that have $p_{12}\geq \lambda$ and $p_{23}\geq \mu$.

It is shown that $f(\lambda,\mu)=0$ when $\lambda +\mu <1$, and $f(1,\mu)=\mu$. Moreover, f($\lambda$,1-$\lambda$) $\leq 1/e$, and, when $\lambda +\mu >1$ and $\max\{\lambda,\mu\}<1$, f($\lambda$,$\mu$) $\leq 1-(1-\lambda)(1-\mu)[1-\log(1-\lambda)(1-\mu)]$. The exact value of f is presently unknown for every case in which $\lambda +\mu \geq 1$ and $\max\{\lambda,\mu\}<1$.

## MSC:

06A06 Partial orders, general



## Keywords:

probabilistic transitivity law; linear extensions; partial order; proportional transitivity law

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