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Quotients by $\mathbb{C}^*$ and SL(2,$\mathbb{C}$) actions. (English) Zbl 0566.32026

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Let the notions be as in the preceding review. The main theorem states the following: There is a bijective correspondence between the set of cross sections (A^+, A^-) and the open sets $U \subset X \setminus X^T$ with the properties: (i) U is T-invariant, (ii) the geometric quotient $U \to U/T$ exists and U/T is a compact complex space. The correspondence is given by $U = \cup_{i\in A^-, j\in A^+} C_{ij}$. Furthermore all such sets U are Zariski open in X. They are called sectional open sets. If X is a projective algebraic variety or a Kähler manifold then $X \setminus X^T$ is the union of all sectional open sets. In the special case that ρ is an algebraic T-action on an algebraic manifold X the following cohomology formula for the quotient of a sectional set U is proved: $P(U/T) = \sum_{i\in A^-} P(F_i)(t^{2d_i^+} - t^{2d_i^-})/t^2 - 1) = \sum_{j\in A^+}(t^{2d_j^-} - t^{2d_j^+})/(t^2 - 1)$, where P denotes the Poincaré polynomial and $d_i^\pm = \dim X_i^\pm - \dim F_i$. Finally the following conjecture of D. Mumford is settled: Consider the diagonal $SL_2(\mathbb{C})$-action on $(\mathbb{P}_1(\mathbb{C}))^n$ (n$\geq$ 3). Let U be a $SL_2(\mathbb{C})$-invariant Zariski open set, which is also invariant under coordinate interchanging. Assume that the geometric quotient $U/SL_2(\mathbb{C})$ exists and is an algebraic variety in the sense of Artin. Then n is odd and U is the set of points with at most (n-1)/2 coordinates the same.

Reviewer: K.Oeljeklaus

MSC:

32M05 Complex Lie groups, group actions on complex spaces
14L30 Group actions on varieties or schemes (quotients)
32J25 Transcendental methods of algebraic geometry (complex-analytic aspects)
32C20 Normal analytic spaces

Keywords:
normal compact space; meromorphic $\mathbb{C}^*$-actions; geometric quotient; sectional open sets

Full Text: DOI