Glasner, Shmuel; **Maon, D.**
An inverted tower of almost 1-1 extensions. (English) Zbl 0567.54026
J. Anal. Math. 44, 67-75 (1985).

In Isr. J. Math. 45, 1-8 (1983; Zbl 0528.28012) the first author and *B. Weiss* have shown that two minimal flows with no common factor need not be disjoint. A question posed by H. Furstenberg was whether two minimal flows exist which have a common almost 1-1 extension (and thus are not disjoint in a very strong sense) and still have no common factor. In what follows we construct a minimal flow (X,T) (X compact metric and $T : X \to X$ a homeomorphism), with two almost 1-1 factors $X \to^{\phi_i} Y_i$ $(i = 1, 2)$, such that there are no minimal flow (Z,T) and homomorphisms $Y_i \to^{\psi_i} Z$ with $\psi_1 \circ \phi_1 = \psi_2 \circ \phi_2$. Choosing any point $x_0 \in X$ we have, in the category of pointed flows, that (X, x_0) is a common almost 1-1 extension of $(Y_1, \phi_1(x_0))$ and $(Y_2, \phi_2(x_0))$ and these latter pointed flows have no non-trivial common pointed factor. This answers a restricted version of Furstenberg's question. We do not have an answer to the original question. The same flow (X,T) also provides an affirmative answer to a question about the existence of an inverted tower of almost 1-1 extensions, namely, there exists a sequence of almost 1-1 homomorphisms

$$X \to^{\psi_1} X_1 \to^{\psi_2} X_2 \to^{\psi_3} \ldots$$

such that for every $x_0 \in X$ the only common pointed factor of the pointed flows $(X_n, \psi_n \circ \psi_{n-1} \circ \ldots \circ \psi_1(x_0))$ is the trivial flow.

MSC:

54H20	Topological dynamics (MSC2010)
28D10	One-parameter continuous families of measure-preserving transformations

Keywords:

minimal flows; common almost 1-1 extension; no common factor; category of pointed flows; inverted tower of almost 1-1 extensions; common pointed factor

Full Text: DOI

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