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The maximum number of cliques in dense graphs. (English) Zbl 0569.05029

Discrete Math. 54, 161-166 (1985).

We will consider only undirected, connected graphs without loops or multiple edges. Denote the number of vertices of G by $|G|$. A clique of graph G is a maximal complete subgraph. The clique graph K(G) of G is the intersection graph of the cliques of G. The density w(G) is the number of vertices in the largest clique of G. A graph is called dense if w(G)≥ $|G|/2$.

This paper makes precise the intuitive idea that very dense graphs have fewer cliques than less dense graphs. First, it is shown that for any graph G, $2^{|G|-w(G)} \geq |K(G)|$. Secondly, this bound is sharp among dense graphs, and among them only. In fact, for all integers s,t≥ 4 where $t \geq s \geq t/2$, there exists a graph G such that $|G| = t$, $w(G) = s$, and $2^{t-s} = |K(G)|$. Call a dense graph packed if $2^{|G|-w(G)} = |K(G)|$. The 2n- Neumann graph is the complement of a matching between 2n vertices. Thirdly, it is shown that any packed graph G contains an induced subgraph isomorphic to the $2[|G| - w(G)]$-Neumann graph. Lastly, the clique graphs of packed graphs are characterized.

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05C35 Extremal problems in graph theory

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