

Fischer-Colbrie, D.

On complete minimal surfaces with finite Morse index in three manifolds. (English)

Zbl 0573.53038

Invent. Math. 82, 121-132 (1985).

The author obtains several good results on the Morse index (for area) of an oriented complete minimal surface (M, g) in a Riemannian 3-manifold N . Amongst them: 1) If $\text{index } M < \infty$, there is a compact $C \subset M$ so that $M \setminus C$ is stable and there is a positive function $u : M \rightarrow \mathbb{R}$ with $Lu = 0$ on $M \setminus C$, where L is the second variation operator. If N has scalar curvature ≥ 0 , then u^2g is a complete metric on M with Gaussian curvature ≥ 0 on $M \setminus C$. In particular, M is conformally equivalent to a Riemann surface with a finite number of punctures. 2) If $N = \mathbb{R}^3$, then $\text{index } M < \infty$ iff M has finite total curvature.

Reviewer: J.Eells

MSC:

53C42 Differential geometry of immersions (minimal, prescribed curvature, tight, etc.)

58E12 Variational problems concerning minimal surfaces (problems in two independent variables)

Cited in **6** Reviews
Cited in **118** Documents

Keywords:

Morse index; minimal surface; second variation operator; total curvature

Full Text: [DOI](#) [EuDML](#)

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