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The Erdős-Dushnik-Miller theorem for topological graphs and orders. (English)

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A topological graph is a graph $G=(V,E)$ on a topological space V such that the edge set E is a closed subset of the product space $V\times V$. If the graph contains no infinite independent set then, by a well-known theorem of Erdős, Dushnik and Miller, for any infinite set $L\subseteq V$, there is a subset L'$\subseteq L$ of the same cardinality $|L'|=|L|$ such that the restriction $G\restriction L'$ is a complete graph. We investigate the question of whether the same conclusion holds if we weaken the hypothesis and assume only that some dense subset $A\subseteq V$ does not contain an infinite independent set. If the cofinality $cf(|L|)>|A|$, then there is an L' as before, but if cf$(|L|)\leq |A|$, then some additional hypothesis seems to be required. We prove that, if the graph $G\restriction A$ is a comparability graph and A is a dense subset, then for any set $L\subseteq V$ such that $cf(|L|)>\omega$, there is a subset L'$\subseteq L$ of size $|L'|=|L|$ such that $G\restriction L'$ is complete. The condition $cf(|L|>\omega$ is needed.

MSC:

05C99 Graph theory
05A05 Permutations, words, matrices
03E05 Other combinatorial set theory

Keywords:

topological graph; topological space; complete graph; independent set; comparability graph

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