Barbu, V.
Optimal control of variational inequalities. (English) ｜Zbl 0574.49005｜
Research Notes in Mathematics, 100. Boston - London - Melbourne: Pitman Advanced Publishing Program. 298 p. £14.95 (1984).

Optimal control of variational inequalities (v.i.) is an area that is currently of considerable interest both from a theoretical and an applications oriented point of view. Much research work has been done during the last couple of years, and there is a constant output of new stimulating results. Therefore, writing a textbook about this subject is not at all an easy task.

The author, who by the way is one of the pioneers in this area, does right that, avoiding to overburden the book, he concentrates on one main aspect, namely the study of first-order necessary optimality conditions for both elliptic and parabolic v.i.'s, while other aspects are only sketched.

The problems are formulated in an abstract Hilbert space setting which makes it possible to apply the powerful apparatus of nonlinear functional analysis, in particular convex analysis. Fundamental results in nonlinear analysis are provided in chapter 1 to make the book self-contained. In chapter 2 existence and regularity results for elliptic v.i.'s are recalled and illustrated by the obstacle problem and the rectangular dam problem. Chapter 3 deals with the maximum principle for such v.i.'s including both the case of distributed and boundary control. The main tool in the analysis is to consider an approximating family of problems regularized by some kind of penalization technique which allows to treat general controls and cost criteria.

After stating some basic results concerning parabolic v.i.'s in chapter 4, the following three chapters are mainly devoted to establishing a Pontryagin type maximum principle for distributed control (chapter 5), boundary control (chapter 6) and time-optimal control (chapter 7) of this kind of v.i.'s by using the techniques developed previously in the stationary case.

Due to the reasons mentioned above this book must be considered as a valuable contribution to optimal control theory and can be recommended not only as a research monograph but also as a class textbook and for self-study.

Reviewer: R.H.W.Hoppe

MSC:

49J40	Variational inequalities
49-02	Research exposition (monographs, survey articles) pertaining to calculus of variations and optimal control
49K20	Optimality conditions for problems involving partial differential equations
35J85	Unilateral problems; variational inequalities (elliptic type) (MSC2000)
35K85	Unilateral problems for linear parabolic equations and variational inequalities with linear parabolic operators
35R35	Free boundary problems for PDEs
49J20	Existence theories for optimal control problems involving partial differential equations

Cited in **5** Reviews
Cited in **212** Documents

Keywords:

Optimal control of variational inequalities; first-order necessary optimality conditions; existence and regularity results; obstacle problem; maximum principle; distributed control; boundary control; time- optimal control