Dihn, Q. V.; **Glowinski, R.**; **Périaux, J.**

Solving elliptic problems by domain decomposition methods with applications. (English)

Zbl 0575.65096

Elliptic problem solvers II, Proc. Conf., Monterey/Calif. 1983, 395-426 (1984).

[For the entire collection see Zbl 0557.00009.]

The contents of this paper are largely included in a longer paper by the same authors [Comput. Methods Appl. Mech. Eng. 40, 27-109 (1983; Zbl 0505.76068)].

The authors are interested in solving elliptic problems using ideas related to the Schwarz alternating method whereby a given domain is broken into several overlapping domains and the Dirichlet problem for Poisson's equation is related to a succession of Dirichlet problems on the smaller domains. This method is suitable for implementation on a parallel (MIMD) computer. In this paper the authors consider both overlapping and non-overlapping domains. In the non-overlapping case they offer two approaches, one involving Neumann conditions on the interfaces and one using Lagrange multipliers to implement Dirichlet conditions. For both cases they describe a conjugate gradient algorithm for the iterative coupling of the domains. They also propose a conjugate gradient variant of Schwarz's method.

The authors present two numerical experiments. The first is the use of the finite element method for a mixed linear elliptic boundary value problem. This problem represents fluid flow in a convergent-divergent duct in two dimensions and is broken into three subdomains. The second is a nonlinear boundary value problem on an infinite domain. This problem represents potential flow around an airfoil in two dimensions. The outer boundary is taken to be at a large distance from the airfoil (rather than at infinity) and the flow domain is broken into an inner and an outer domain. The inner domain is solved using a finite element method, the outer domain is solved using exponential stretching coupled with a finite difference method. The domains are coupled using a conjugate gradient algorithm preconditioned by a natural elliptic operator.

Reviewer: M.Sussman

MSC:

65N22	Numerical solution of discretized equations for boundary value problems involving PDEs
65F10	Iterative numerical methods for linear systems
35J25	Boundary value problems for second-order elliptic equations
35J65	Nonlinear boundary value problems for linear elliptic equations
76B10	Jets and cavities, cavitation, free-streamline theory, water-entry problems, airfoil and hydrofoil theory, sloshing
76B15	Water waves, gravity waves; dispersion and scattering, nonlinear interaction
76H05	Transonic flows

Keywords:

domain decomposition; Schwarz alternating method; Poisson's equation; Lagrange multipliers; conjugate gradient algorithm; numerical experiments; finite element method; exponential stretching; finite difference method