Dreesen, B.; **Poguntke, W.**; **Winkler, P.**
Comparability invariance of the fixed point property. (English) Zbl 0577.06005
Order 2, 269-274 (1985).

The comparability graph of an ordered set P is the undirected graph whose vertex set is P and in which two vertices x,y are adjacent if and only if $x < y$ or $x > y$. A property of an ordered set is called comparability invariant, if two arbitrary ordered sets with isomorphic comparability graphs either both have this property, or both have not. If P, Q are ordered sets and $a \in P$, then P(a,Q) is the ordered set obtained from P by replacing a by Q. An ordered set P is said to have the fixed point property, if every order-preserving mapping from P to P has at least one fixed point.

Three theorems are proved. The first of them has an auxiliary character; the further ones are the following. Theorem 2. Let P and Q be finite ordered sets with $a \in P$. Then P(a,Q) has the fixed point property if and only if either (i) P and Q both have the fixed point property; or (ii) P has the fixed point property, Q does not, and there is no order- preserving map $f : P \to P$ which fixes only a and sends no element above or below a to a. Theorem 3. The fixed point property is comparability invariant for finite ordered sets.

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