Nevesenko, N. V.; **Oshman, E. V.**

Metric projection onto convex sets. (English. Russian original) Zbl 0578.41038

Math. Notes 31, 59-64 (1982); translation from Mat. Zametki 31, 117-126 (1982).

Let X be a real linear normed space and let X^* be its conjugate space. For any nonempty sets M, N in X, define $d(x,M) = \inf\{\|x-y\| : \quad y \in M\}$ and $d(M,N) = \sup\{d(x,N) : \quad x \in M\}$. The set-valued mapping $P_M x = \{y \in M : \quad \|x-y\| = d(x,M)\}$ is called the metric projection from X onto M. A set M in X is called a strict convex subset, if M is convex, closed, has nonempty interior, and its boundary contains no interval. A real linear normed space X is called (RBℝ) space (or $X \in (RBR)$), if for every $f \in X^*$, $\|f\| = 1$, $\Gamma_f = \{x : f(x) = \|x\| = 1\}$ is either empty, or singleton, or a strict convex subset in the hyperplane $\{x \in X : f(x) = 1\}$. A metric projection P_M is called lower semi-continuous if for any $x \in X$, $y \in P_M x$ and $x_n \to x$, there holds $d(y, P_M x_n) \to 0$ is called lower H-semi-continuous if for any $x \in X$ and $x_n \to x$ there holds $d(P_M x, P_M x_n) \to 0$.

In this paper the authors study the relations of the various continuous properties of metric projection and the structure of Banach space. The main result is the following: Theorem 4. For Banach space X, the following statements are mutually equivalent: (1) $X \in (RBR)$; (2) for any 3-dimensional subspace X_3, of X, $X_3 \in (RBR)$; (3) for any 3-dimensional subspace X_3 of X the metric projection from X onto any closed convex subset of X_3 is lower semi-continuous; (4) the metric projection from X onto any bounded compact convex subset $M \subset X$ is lower semi-continuous; (5) the metric projection from X onto any bounded compact convex subset $M \subset X$ lower H-semi-continuous.

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MSC:

41A65 Abstract approximation theory (approximation in normed linear spaces and other abstract spaces)

41A50 Best approximation, Chebyshev systems

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