

**Freund, Robert M.; Orlin, James B.**

**On the complexity of four polyhedral set containment problems.** (English) Zbl 0581.90060  
*Math. Program.* 33, 139-145 (1985).

A nonempty closed convex polyhedron  $X$  can be represented either as  $X = \{x : Ax \leq b\}$ , where  $(A, b)$  are given, in which case  $X$  is called an H-cell, or in the form  $X = \{x : x = U\lambda + V\mu, \sum \lambda_j = 1, \lambda \geq 0, \mu \geq 0\}$ , where  $(U, V)$  are given, in which case  $X$  is called a W-cell. This note discusses the computational complexity of certain set containment problems. The problems of determining if  $X \not\subseteq Y$ , where (i)  $X$  is an H-cell and  $Y$  is a closed solid ball, (ii)  $X$  is an H-cell and  $Y$  is a W-cell, or (iii)  $X$  is a closed solid ball and  $Y$  is a W-cell, are all shown to be NP-complete, essentially verifying a conjecture of *B. C. Eaves* and the first author [ibid. 23, 138-147 (1982; [Zbl 0479.90064](#))]. Furthermore, the problem of determining whether there exists an integer point in a W-cell is shown to be NP-complete, demonstrating that regardless of the representation of  $X$  as an H-cell or W-cell, this integer containment problem is NP-complete.

**MSC:**

[90C10](#) Integer programming  
[52Bxx](#) Polytopes and polyhedra  
[68Q25](#) Analysis of algorithms and problem complexity

Cited in **1** Review  
Cited in **28** Documents

**Keywords:**

H-cell; W-cell; computational complexity; set containment problems; NP- complete

**Full Text:** [DOI](#)

**References:**

- [1] B.C. Eaves and R.M. Freund, "Optimal scaling of balls and polyhedra", *Mathematical Programming* 23 (1982) 138–147. · [Zbl 0479.90064](#) · [doi:10.1007/BF01583784](#)
- [2] F.R. Gantmacher, *Matrix theory*, vol. 1 (Chelsea, New York, 1959). · [Zbl 0085.01001](#)
- [3] M.R. Garey and D.S. Johnson, *Computers and intractability* (W.H. Freeman, San Francisco, 1979).
- [4] R.M. Karp, "Reducibility among combinatorial problems", in: R.E. Miller and J.W. Thatcher, eds., *Complexity of computer computations* (Plenum Press, New York, 1972) pp. 85–103.

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