Freund, Robert M.; Orlin, James B.
On the complexity of four polyhedral set containment problems. (English) Zbl 0581.90060
Math. Program. 33, 139-145 (1985).

A nonempty closed convex polyhedron X can be represented either as $X=\{x: \mathrm{Ax}\leq b\}$, where (A,b) are given, in which case X is called an H-cell, or in the form $X=\{x: x=U\lambda+V\mu, \sum\lambda_j=1, \lambda\geq 0, \mu\geq 0\}$, where (U,V) are given, in which case X is called a W-cell. This note discusses the computational complexity of certain set containment problems. The problems of determining if $X\not\subseteq Y$, where (i) X is an H-cell and Y is a closed solid ball, (ii) X is an H-cell and Y is a W-cell, or (iii) X is a closed solid ball and Y is a W-cell, are all shown to be NP-complete, essentially verifying a conjecture of *B. C. Eaves* and the first author [ibid. 23, 138-147 (1982; Zbl 0479.90064)]. Furthermore, the problem of determining whether there exists an integer point in a W-cell is shown to be NP-complete, demonstrating that regardless of the representation of X as an H-cell or W-cell, this integer containment problem is NP-complete.

MSC:

90C10 Integer programming
52Bxx Polytopes and polyhedra
68Q25 Analysis of algorithms and problem complexity

Keywords:

H-cell; W-cell; computational complexity; set containment problems; NP- complete

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