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Convergence of solutions of H-systems or how to blow bubbles. (English) Zbl 0584.49024

Arch. Ration. Mech. Anal. 89, 21-56 (1985).

The authors study the dependence of parametric surfaces of constant mean curvature on the boundary curve when the boundary curve degenerates to one point. They look at the normalized situation where the surface $u=(u_1,u_2,u_3)$, $u=u(x,y)$ is parametrized on the unit disk B in $\mathbb{R}^2$ and possesses mean curvature 1. The surface is given in conformal parameters and the boundary condition is of Plateau type, i.e. u maps ∂B onto some Jordan curve Γ in $\mathbb{R}^3$. Now let Γ_n be a sequence of Jordan curves with $\Gamma_n \to 0$ (n$\to \infty$) and let u^n be surfaces as above, bounded by Γ_n, with uniformly bounded area. Then it is proved that a subsequence of u_n converges to 0 or to a finite union of spheres of radius one. If one takes as u^n large solutions of the problem then a subsequence converges to a single sphere of radius one containing 0.

The proofs rely on a blow up argument which leads to the system $\Delta w = 2w_x \bigwedge w_y$ on $\mathbb{R}^2$ with finite Dirichlet integral. A careful analysis shows that w is the stereographic projection of a rational function on $\mathbb{C}$. Dirichlet's integral is in $8\pi\mathbb{Z}^+$.

The main result is that any sequence of solutions of the Dirichlet problem $\Delta u^n = 2u^n_x \bigwedge u^n_y$ on B, $u^n=\gamma^n$ on ∂B with $\gamma^n \to 0$ (n$\to \infty$), bounded in $H^{1,2}(B,\mathbb{R}^3)$ after a blow up essentially behaves like a finite superposition of solutions w as above.

Reviewer: G.Dziuk

MSC:

49Q05	Minimal surfaces and optimization
35J60	Nonlinear elliptic equations
53A10	Minimal surfaces in differential geometry, surfaces with prescribed mean curvature

Cited in **3** Reviews
Cited in **137** Documents

Keywords:

degenerate boundary curve; big solutions; parametric surfaces of constant mean curvature; Dirichlet's integral

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