

Silverman, Joseph H.

The arithmetic of elliptic curves. (English) Zbl 0585.14026

Graduate Texts in Mathematics, 106. New York etc.: Springer-Verlag. XII, 400 p. DM 148.00 (1986).

This book provide a nice introduction to a presently very active part of mathematics.

Its first part gives the fundamental notions, beginning by short chapters reviewing the main definitions and results for algebraic varieties, algebraic curves (including Riemann-Roch theorem); it goes on with the basic definitions for elliptic curves (Weierstraß models, the group law, isogenies, Tate module...) and their formal group.

The chapter V studies elliptic curves on finite fields; after proving the Weil conjectures for them, it discusses the Hasse invariant and supersingular curves. Chapter VI studies elliptic curves over \mathbb{C} using the Weierstraß \wp -function; the structure of the group of torsion is given. The chapter ends by a discussion of the Lefschetz principle allowing to extend results to other fields of characteristics 0. The chapter VII studies elliptic curves over a local field: it discusses the minimal model, and gives the Néron-Ogg-Shafarevich criterion for good reduction. With the chapter VIII begins the theory of elliptic curves over a global field. Its purpose is to prove the Mordell-Weil theorem (The group of rational points of E over K is of finite type) using descent theory and heights. The case where $K = \mathbb{Q}$ is settled in first by down to earth considerations. Mestre's example of an elliptic curve of rank 12 is given. Chapter IX studies the integral points using Siegel's theorem. It also discusses effective problems with the help of Baker's theory. It proves the Shafarevich theorem about the finiteness of the number of elliptic curves with good reduction outside a given set of primes. Chapter X (the summit of the book) discusses descent theory to compute the Mordell-Weil group. It defines the Selmer group and the Shafarevich groups. It ends with a discussion about the famous family $y^2 = x^3 - 4dx$.

The book ends with 3 appendices; the first is about characteristics 2 and 3, the second is about Galois theory, the third is a brief summary of other important questions (the contents of a second volume ?). Note that this book has a good index and gives an important bibliography.

I think that this book will be very useful for people beginning in the field of research about elliptic curves.

Reviewer: [R. Gillard](#)

MSC:

- 11G05 Elliptic curves over global fields
- 11-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to number theory
- 11G07 Elliptic curves over local fields
- 11G20 Curves over finite and local fields
- 11G40 L -functions of varieties over global fields; Birch-Swinnerton-Dyer conjecture
- 14H45 Special algebraic curves and curves of low genus
- 14G05 Rational points
- 14H25 Arithmetic ground fields for curves
- 14-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to algebraic geometry
- 14H52 Elliptic curves
- 11D25 Cubic and quartic Diophantine equations

Cited in **15** Reviews
Cited in **839** Documents

Keywords:

elliptic curves over finite fields; Weil conjectures; group of torsion; elliptic curves over local fields; good reduction; elliptic curves over global fields; Mordell-Weil theorem; descent; Selmer group; Shafarevich groups