

**Oshman, E. V.**

**Continuity of metric projection.** (English. Russian original) Zbl 0586.41026

*Math. Notes* 37, 114-119 (1985); translation from *Mat. Zametki* 37, No. 2, 200-211 (1985).

Continuing his previous investigation, the author gives new geometric characterizations of the continuity of the metric projection  $P_M$ , for all  $M$  in a given class  $\mathfrak{M}$  of subsets of a Banach space  $X$  (for example,  $\mathfrak{M}$  may be the class of all weakly compact convex subsets of  $X$ , the class of all boundedly weakly compact convex subsets of  $X$ , etc.). The main result of the paper is Theorem 2: For a Banach space  $X$  the following conditions are equivalent: 1)  $X \in (A_\partial) \cap (RBR)$ ; 2)  $P_M$  is Hausdorff continuous,  $M \in \mathfrak{M}$ ; 3)  $P_M$  is Hausdorff lower semicontinuous,  $M \in \mathfrak{M}$ ; 4)  $P_M$  is lower semicontinuous,  $M \in \mathfrak{M}$ . Condition 1) is given in terms involving faces of the unit sphere  $S$  of  $X$  and weak and norm convergence of sequences in  $S$ .

Reviewer: [S.Cobzaş](#)

**MSC:**

[41A65](#) Abstract approximation theory (approximation in normed linear spaces and other abstract spaces)

[46B20](#) Geometry and structure of normed linear spaces

Cited in **1** Review  
Cited in **4** Documents

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