Besson, Gérard

On symmetrization. (English) Zbl 0589.53049

Nonlinear problems in geometry, Proc. AMS Spec. Sess., 820th Meet. AMS, Mobile/Ala. 1985, Contemp. Math. 51, 9-21 (1986).

[For the entire collection see Zbl 0579.00012.]

A very important problem in Riemannian geometry is to find answers to the following question: To what extent do estimates on the curvature of a Riemannian manifold M enforce global restrictions on the manifold ? The author studies some partial results coming from analytic methods, and in particular those involving linear analysis on Hilbert spaces, rather than the partial differential aspects of the problem. The starting point of the author's study of this problem was the following theorem: Let Ric be the Ricci curvature tensor of M and define for $m \in M$

$$r(m) = \inf\{Ric_m(u_m, u_m) : \ u_m \text{ a unit tangent vector at } m\}, \text{ and } r_{\min} = \inf\{r(m) : m \in M\}.$$

Also let d be the diameter of M.

Theorem [*P. Bérard* and *S. Gallot*, Sémin. Goulaouic-Meyer- Schwartz, Equations Dériv. Partielles 1983-1984, Exp. No.15, 34 p. (1984; Zbl 0542.53025)]. With the above notations, if $\alpha \in R_+$ and $\varepsilon \in \{-1, 0, 1\}$ are such that $r_{\min} d^2 \geq \varepsilon(n-1)\alpha^2$, then

$$\mathrm{tr}(e^{-t\Delta_M}) = Z_M(t) \leq Z_{S^n}(t/R^2) = Z_{S^n(R)}(t)$$

where S^n is the canonical n-sphere, $S^n(R)$ is the n-sphere of radius R, and $R = d/\alpha(n, \varepsilon, \alpha)$ where $\alpha(n, \varepsilon, \alpha)$ is a number depending explicitly on n, ε and α only.

Reviewer: T.Rassias

MSC:

53C20 Global Riemannian geometry, including pinching
58J50 Spectral problems; spectral geometry; scattering theory on manifolds
53C35 Differential geometry of symmetric spaces

Keywords:
rough Laplacian