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Harmonic and holomorphic maps. (English) Zbl 0591.53031

Geometry Semin. "Luigi Bianchi", Lect. Sc. Norm. Super., Pisa 1984, Lect. Notes Math. 1164, 161-224 (1985).

[For the entire collection see Zbl 0568.00011.]

This article is based on a series of seminars given at the Scuola Normale which relate the theories of harmonic maps between Riemannian manifolds and holomorphic maps between almost complex manifolds using twistor spaces. The author begins by explaining the result, due to Lichnerowicz, that a holomorphic map $\phi : M \to N$ between almost Hermitian manifolds is harmonic if M is cosymplectic (i.e. $d^*\omega_M = 0$ where ω_M is the fundamental 2-form) and N is (1,2)-symplectic (i.e. the (1,2)-component of $d\omega_N$ is zero). This is generalized by allowing N to be any oriented even dimensional Riemannian manifold with twistor space $\pi : S \to N$. There is a non-integrable almost complex structure on S such that if $\psi : M \to S$ is holomorphic then $\pi \circ \psi$ is harmonic. Consideration of the case when M is a Riemann surface leads to a characterization of minimal surfaces in N in terms of holomorphic curves in S (in particular when N is a sphere). When N is a symmetric space S can be replaced by subbundles which are twistor spaces in a generalized sense. These techniques are used to prove results due to several authors on harmonic maps into complex projective spaces, quaternionic Kähler manifolds, Gauss maps, CR manifolds, minimal surfaces in S^4 and holomorphic curves in S^6.

Reviewer: F.Kirwan

MSC:

53C15	General geometric structures on manifolds (almost complex, almost product structures, etc.)	Cited in **2** Reviews Cited in **20** Documents
58E20	Harmonic maps, etc.	
32Q99	Complex manifolds	
53C55	Global differential geometry of Hermitian and Kählerian manifolds	
53C20	Global Riemannian geometry, including pinching	

Keywords:

harmonic maps; holomorphic maps; almost complex manifolds; twistor spaces; minimal surfaces; holomorphic curves; symmetric space; complex projective spaces; quaternionic Kähler manifolds; Gauss maps; CR manifolds

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