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A Lusin type approximation of Sobolev functions by smooth functions. (English)

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[For the entire collection see Zbl 0565.00007.]

This paper is concerned with the approximation of a Sobolev function f by a smooth function g in such a way that $|f-g|$ is small and the set where f and g disagree has small capacity. The following is the main theorem. Let $1 \leq p < \infty$, let ℓ, m be positive integers, with $1 \leq m \leq \ell$ and $(\ell - m)p < n$ and let Ω be a non-empty open set of R^n. Let $f \in W^{\ell,p}(\Omega)$ and be approximately continuous at each point of Ω except for a set E with Riesz capacity $R_{\ell-m,p}(E) = 0$. Let $\epsilon > 0$. Then there exists a C^m function g on Ω, such that (a) the set $F = \{x; x \in \Omega$ and f(x)$\neq g(x)\}$ has $R_{\ell-m,p}(F) < \epsilon$ and (b) $|f-g|_{m,h} < \epsilon$.

In another theorem it is shown that each $f \in W^{\ell,p}(\Omega)$ can be represented by a function which is approximately continuous except for a set E, with $R_{\ell-m,p}(E) = 0$. Since $R_{0,p}$ is equivalent to Lebesgue measure, the above theorem generalises a result of *Fon-Che Liu* [Indiana Univ. Math. J. 26, 645-651 (1977; Zbl 0368.46036)].

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Sobolev function; smooth function; small capacity; Riesz capacity